Seminar 3 Serii Fourier

Rezolvare temă

1). $x(t) = e^{-2t}$

$T = 4 \quad t \in (-2, 2)$

$$c_k = \frac{1}{T} \int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$\Rightarrow c_k = \frac{1}{4} \int_{-2}^{2} e^{-2t} e^{-jk\frac{\pi}{2}t} dt =$$

$$= \frac{1}{4} \int_{-2}^{2} e^{-(2+jk\frac{\pi}{2})t} dt =$$

$$= -\frac{1}{4(2+jk\frac{\pi}{2})} e^{-(2+jk\frac{\pi}{2})t} \Big|_{-2}^{2}$$

$$= -\frac{1}{4(2+jk\frac{\pi}{2})} \cdot \left[e^{-(2+jk\frac{\pi}{2})2} - e^{(2+jk\frac{\pi}{2})\cdot 2}\right]$$

$$= -\frac{1}{4(2+jk\frac{\pi}{2})} \left[e^{-4} \cdot e^{-jk\pi} - e^{4} e^{jk\pi}\right]$$

$$= -\frac{(-1)^k}{4(2+jk\frac{\pi}{2})} (e^{-4} - e^{4}) = \frac{(-1)^{k+1}[e^{-4} - e^{4}]}{2(4+jk\pi)}$$

$$e^{-jk\pi} = \cos k\pi - j \sin k\pi = (-1)^k - j \cdot 0 = (-1)^k$$

$$c_k = \frac{(-1)^{k+1}}{2(16+k^2\pi^2)} [e^{-4} - e^{4}](4 - jk\pi)$$

$$c_k = \frac{(-1)^k [e^{4} - e^{-4}]}{2(16+k^2\pi^2)} (4 - jk\pi)$$

$$|c_k| = \frac{[e^4 - e^{-4}]}{2(16+k^2\pi^2)} \sqrt{16+k^2\pi^2}$$

$$\begin{cases} |c_k| = \dfrac{e^4 - e^{-4}}{2\sqrt{16+k^2\pi^2}} \\ \arg\{c_k\} = \operatorname{arctg}\left(-\dfrac{k\pi}{4}\right) = -\operatorname{arctg}\left(\dfrac{k\pi}{4}\right) \end{cases}$$

2) $x[n] = e^{-2n}$ $\quad n \in \{-2,-1,0,1\}$ $\quad N=4.$

$$\Omega_0 = \frac{2\pi}{N} = \frac{\pi}{2}.$$

$$c_k = \frac{1}{N}\sum_{n=-2}^{1} x[n]\cdot e^{-jk\frac{\pi}{2}n} = \frac{1}{4}\sum_{-2}^{1} x[n] e^{-jk\frac{\pi}{2}n}$$

$$c_k = \frac{1}{4}\left[e^4 e^{jk\pi} + e^2 e^{jk\frac{\pi}{2}} + 1 + e^{-2} e^{-jk\frac{\pi}{2}}\right]$$

$$e^{jk\frac{\pi}{2}} = \begin{cases} 1 & k=4p \\ j & k=4p+1 \\ -1 & k=4p+2 \\ -j & k=4p+3 \end{cases} \qquad e^{-jk\frac{\pi}{2}} = \begin{cases} 1 & k=4p \\ -j & k=4p+1 \\ -1 & k=4p+2 \\ j & k=4p+3 \end{cases}$$

$\boxed{k=4p}$

$$\Rightarrow c_{4p} = \frac{1}{4}\left[e^4 + e^2 + 1 + e^{-2}\right] = \frac{(e^4+1)(1+e^{-2})}{4}.$$

$$e^2 + e^{-2} = e^{-2}(e^4+1)$$

$\boxed{k=4p+1}$

$$c_{4p+1} = \frac{1}{4}\left[-e^4 + je^2 + 1 - je^{-2}\right] = \frac{e^4-1}{4}\left[-1+je^{-2}\right]$$

$$|c_{4p+1}| = \frac{e^4-1}{4}\sqrt{1+e^{-4}}$$

$$\arg c_{4p+1} = \pi - \operatorname{arctg}(e^{-2})$$

$\boxed{k=4p+2}$ $\quad c_{4p+2} = \frac{1}{4}\left[+e^4 - \cancel{j}e^2 + 1 - e^{-2}\right] = \frac{e^4+1}{4}\left[1-e^{-2}\right]$

$\boxed{k=4p+3}$

$$c_{4p+3} = \frac{1}{4}\left[-e^4 - je^2 + 1 + je^{-2}\right] = \frac{e^4-1}{4}\left[-1+je^{-2}\right]$$

$$|c_{4p+3}| = \frac{e^4-1}{4}\sqrt{1+e^{-4}} \qquad \arg c_{4p+3} = \pi - \operatorname{arctg} e^{-2}$$