Sisteme

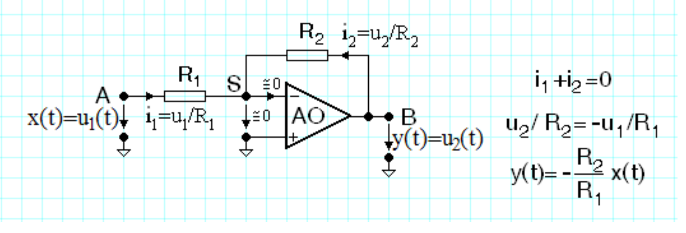
Semnalele pot fi supuse prelucrarii in scopul obtinerii unor alte semnale, sau al obtinerii unor parametri ai acestora.

Prelucrarile se aplica unui semnal de intrare x(t) si se obtine un alt semnal, de iesire, y(t).

Modularea/demodularea, filtrarea etc.

http://shannon.etc.upt.ro/teaching/ssist/Cap2.pdf

Sistem analogic



amplificator

$A_u > 10000$

$\frac{u_2}{V_S} = A_u$;

$V_S = \frac{u_2}{A_u} < \frac{u_2}{10000}$

$|u_2| < 5V$;

$|V_S| < 500\ \mu V$;

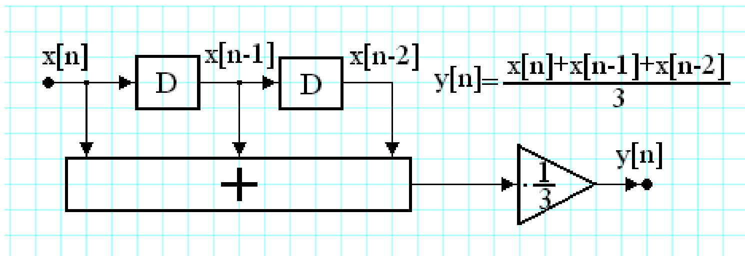
$V_S = 0$

$i_i = \frac{V_S}{R_{in}}$;

$R_{in} = 100\ k\Omega$;

$i_i < \frac{500\ \mu V}{100\ k\Omega} = 5\ nA$

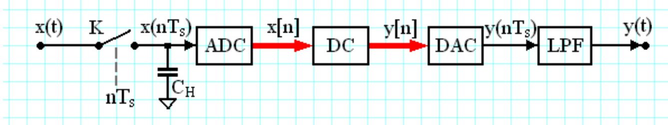
Sistem digital

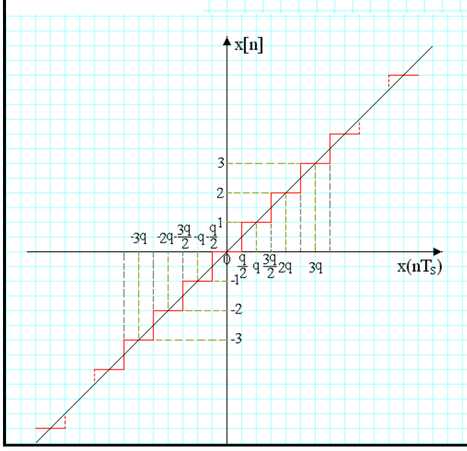


Mediere alunecătoare.

Algoritm.

Simularea sistemului analogic folosind unul digital





$x(nT_s) \cong qx[n]$

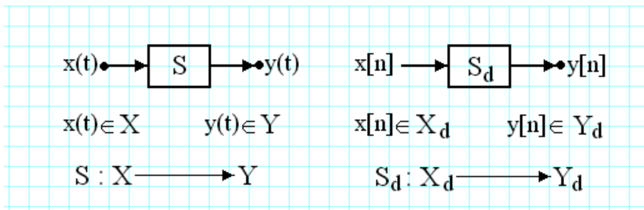
convertor pe 10 biti (1024 nivele de cuantizare)

domeniu al tensiunii de intrare 10V

$q \cong \frac{10\text{V}}{1024} \cong 10\text{mV}$

Eroarea absoluta maxima $\pm 5\text{mV}$

Modelul matematic



$$y(t)=S\{x(t)\} \quad \text{sau} \quad x(t)\overset{S}{\rightarrow}y(t);\; y[n]=S_d\{x[n]\} \quad \text{sau} \quad x[n]\overset{S_d}{\rightarrow}y[n]$$

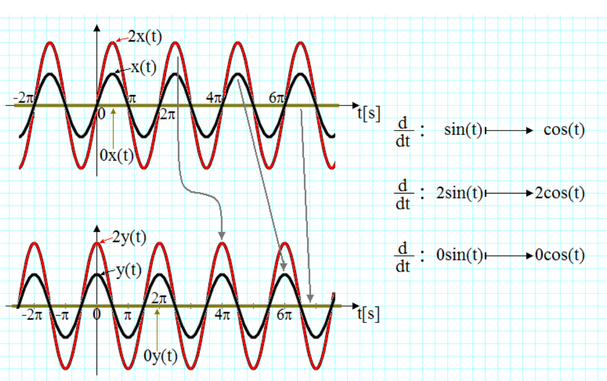
Sunt modelate prin operatori.

$$\frac{d}{dt} \quad : \quad x(t)\rightarrow x'(t)$$

$$\int \quad : \quad x(t) \rightarrow \int_{-\infty}^{t} x(\tau)d\tau$$

Sisteme liniare

$$S\{a_1x_1(t)+a_2x_2(t)\}=a_1S\{x_1(t)\}+a_2S\{x_2(t)\}$$
$$S_d\{a_1x_1[n]+a_2x_2[n]\}=a_1S_d\{x_1[n]\}+a_2S_d\{x_2[n]\}$$



Omogenitate

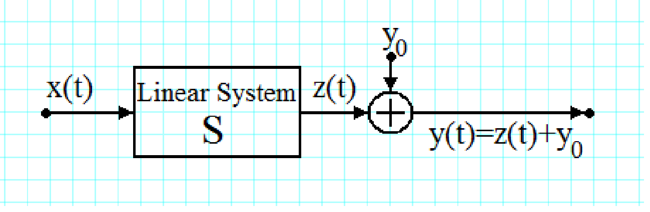
$$S\{ax(t)\}=aS\{x(t)\}$$
$$S_d\{ax[n]\}=aS_d\{x[n]\}$$

Sistem incremental liniar

Pentru un sistem omogen, $a = 0$: $S\left\{0x(t)\right\} = 0S\left\{x(t)\right\} = 0$

Sisteme cu variatii la iesire proportionale cu variatiile de la intrare.

Se modeleaza printr-un sistem liniar, la iesirea caruia se adauga valoarea de repaus (zero-input response) y_0



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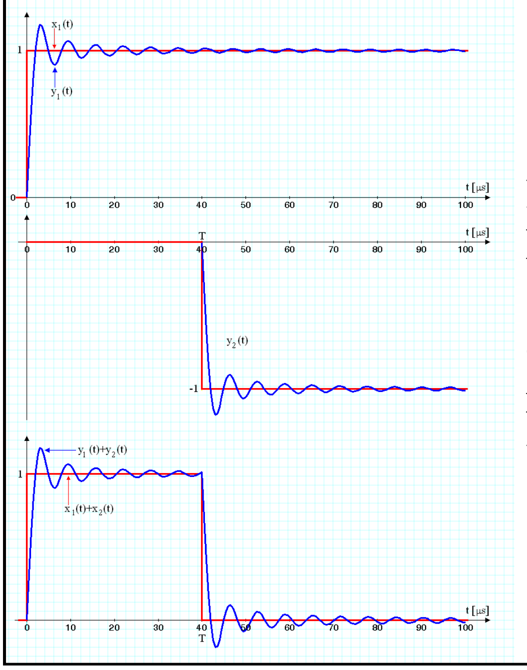

Aditivitatea

Răspunsul sistemului liniar la suma a doua semnale de intrare este suma răspunsurilor la fiecare semnal.

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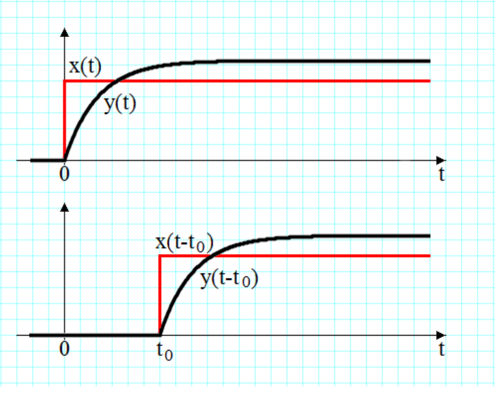
Sisteme invariante la translaţia în timp



$$S\{x(t)\} = y(t) \quad \Rightarrow$$
$$S\{x(t-t_0)\} = y(t-t_0)$$

Sistem liniar si invariant in timp = SLIT (linear time-invariant system, LTI)

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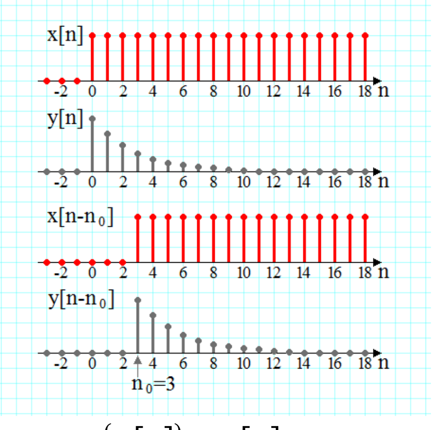

$$S_d\{x[n]\} = y[n] \Rightarrow$$
$$S_d\{x[n-n_0]\} = y[n-n_0]$$

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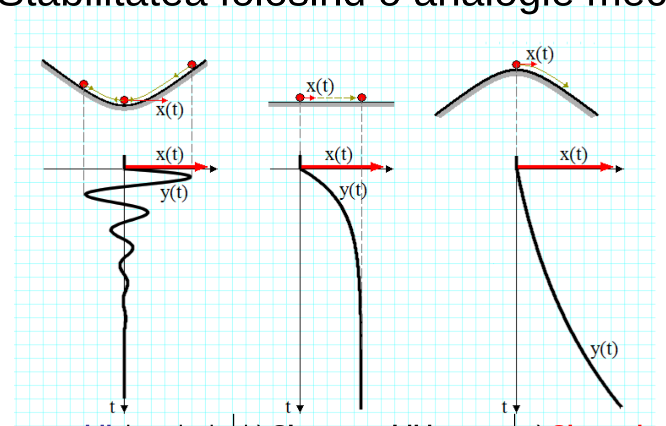
Stabilitatea folosind o analogie mecanica



a) **Sistem stabil**: impulsul aplicat bilei determina oscilatii ale pozitiei sale, care se amortizeaza –bila revine in pozitia de echilibru initial.

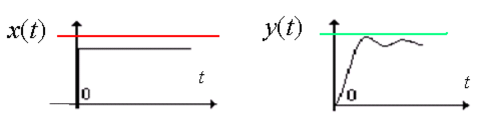
b) **Sistem stabil la limita**: impulsul aplicat bilei modifica pozitia de echilibru.

c) **Sistem instabil**: impulsul aplicat bilei duce la pierderea echilibrului.

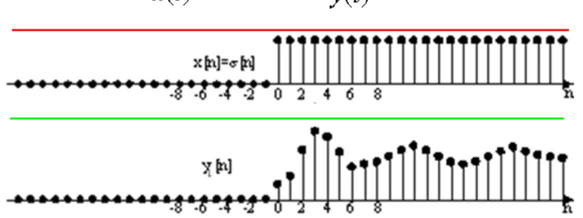
BIBO Stabilitatea sistemelor

BIBO- bounded input bounded output

Dacă semnalul de intrare este mărginit şi semnalul de ieşire trebuie să fie mărginit.

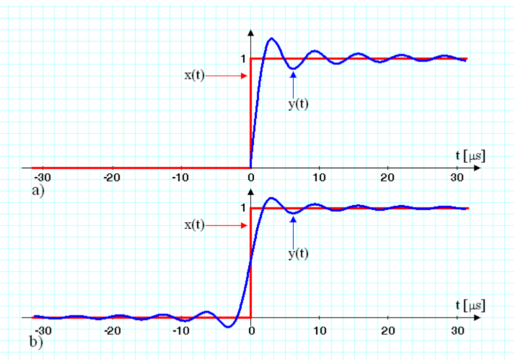


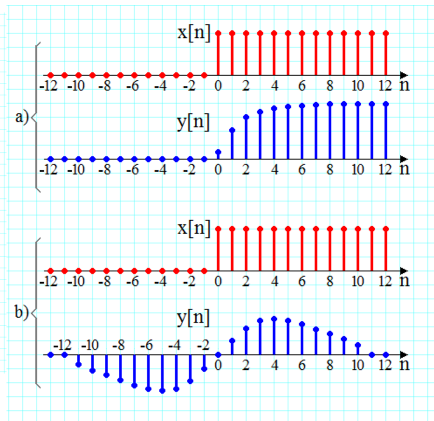
$x(t) \overset{S}{\longmapsto} y(t)$



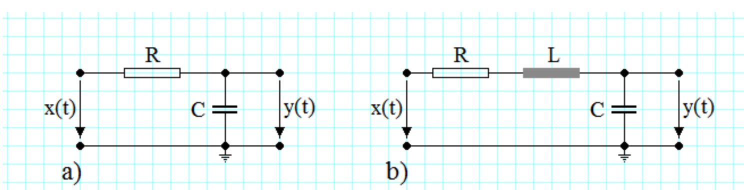
Cauzalitatea sistemelor

Efectul să nu apară înaintea cauzei.





2.6. Sisteme analogice descrise prin ecuatii diferentiale si sisteme digitale descrise prin ecuatii cu diferente finite, cu coeficienti constanti



Sistem liniar R-C de ordin unu.

$$RC\frac{dy(t)}{dt}+y(t)=x(t)$$

Sistem liniar R-L-C de ordin doi.

$$LC\frac{d^2y(t)}{dt^2}+RC\frac{dy(t)}{dt}+y(t)=x(t)$$

Tema de casa: Sa se demonstreze liniaritatea acestor sisteme.

Forma generala a ecuatiei diferentiale care descrie un sistem liniar de ordin N

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}, \quad a_N \neq 0 \text{ (macar)}, \quad a_k, b_k = \text{const.}$$

Conditiile initiale trebuie sa fie nule daca sistemul este liniar, adica:

$$y(t_0) = \left.\frac{dy(t)}{dt}\right|_{t=t_0} = \left.\frac{d^2 y(t)}{dt^2}\right|_{t=t_0} = \ldots = \left.\frac{d^{N-1} y(t)}{dt^{N-1}}\right|_{t=t_0} = 0$$

daca momentul de aplicare al semnalului de intrare este t_0

$x(t) \equiv 0$ pentru $t < t_0$

Un sistem digital echivalent sistemului de ordin unu

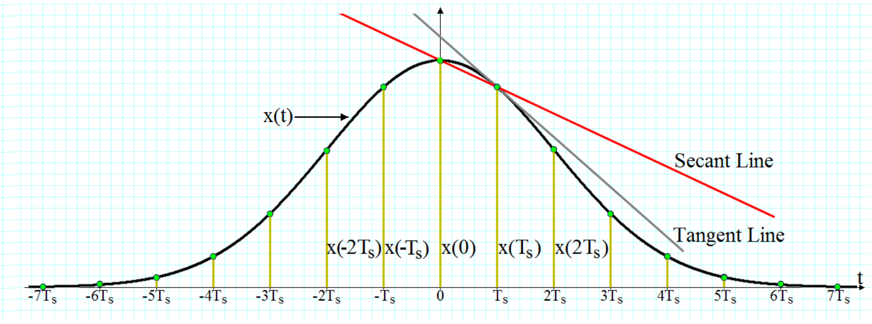
$$RC \frac{dy(t)}{dt} + y(t) = x(t)$$

$RC \left.\frac{dy(t)}{dt}\right|_{t=nT_e} + y(nT_e) = x(nT_e)$ - Derivata poate fi aproximata:

$$\left.\frac{dy(t)}{dt}\right|_{t=nT_e} \cong \frac{y(nT_e) - y(nT_e - T_e)}{T_e} = \frac{y[n] - y[n-1]}{T_e}$$

$$\left(\frac{RC}{T_e} + 1\right) y[n] - \frac{RC}{T_e} y[n-1] = x[n]$$

Ecuatie cu diferente finite, cu coeficienti constanti, obtinuta prin aproximarea ecuatiei diferentiale.



- Panta secantei este o aproximare buna pentru panta tangentei daca se considera o valoare mica a pasului de esantionare T_e

Un sistem digital echivalent sistemului de ordin doi

$$LC\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT_e}+RC\left.\frac{dy(t)}{dt}\right|_{t=nT_e}+y(nT_e)=x(nT_e)$$

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT_e}=\frac{d}{dt}\left.\left(\frac{dy(t)}{dt}\right)\right|_{t=nT_e}\cong\frac{\left.\frac{dy(t)}{dt}\right|_{t=nT_e}-\left.\frac{dy(t)}{dt}\right|_{t=nT_e-T_e}}{T_e}$$

$$=\frac{\frac{y[n]-y[n-1]}{T_e}-\frac{y[n-1]-y[n-2]}{T_e}}{T_e}=\frac{y[n]-2y[n-1]+y[n-2]}{T_e^2}$$

$$\left(\frac{LC}{T_e^2}+\frac{RC}{T_e}+1\right)y[n]-\left(\frac{2LC}{T_e^2}+\frac{RC}{T_e}\right)y[n-1]+\frac{LC}{T_e^2}y[n-2]=x[n]$$

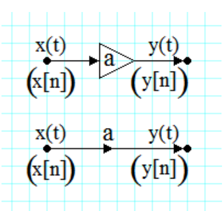
Ecuatie cu diferente finite, cu coeficienti constanti, obtinuta prin aproximarea ecuatiei diferentiale.

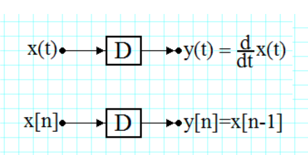
Forma generala a ecuatiei cu diferente finite care descrie un sistem digital liniar de ordin N

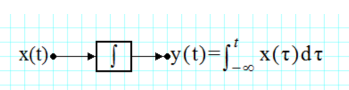
$$\sum_{k=0}^{N} A_k y[n-k] = \sum_{k=0}^{M} B_k x[n-k], \quad A_N \neq 0 \text{ (macar)}$$

$$A_k, B_k = \text{const.}$$

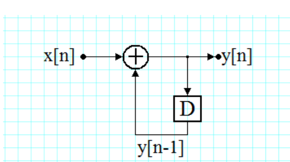
Exemple de sisteme şi simboluri folosite







*Circuite Integrate Analogice



Simbolul folosit pentru sistemul proportional ideal

Simbolul folosit pentru sistemul diferentiator ideal

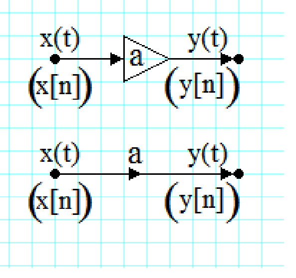
Simbolul folosit pentru sistemul integrator ideal

2.7. Câteva exemple de sisteme

i) Sistemul proportional ideal

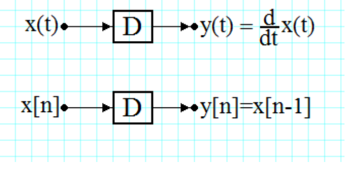
$$y(t)=ax(t), \quad a\in\mathbb{R} \qquad\qquad y[n]=ax[n], \quad a\in\mathbb{R}$$

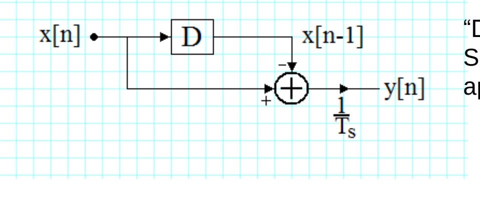
Sistem fara memorie: Valoarea curenta a semnalului de iesire depinde doar de valoarea curenta a semnalului de la intrare, nu si de valorile sale anterioare.



ii) Sistemul ideal de derivare (analogic) si de intarziere (digital)

$$y(t)=\frac{dx(t)}{dt} \qquad y[n]=\frac{1}{T_S}\left(x[n]-x[n-1]\right)$$





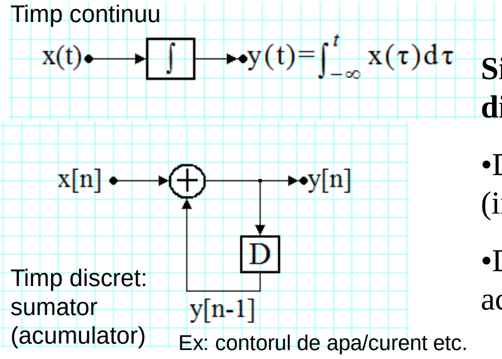
“Diferentierea” finita.
Sistem discret ce implementeaza aproximarea derivatei din timp continuu

iii) Sistemul integrator ideal

$$y(t)=\int_{-\infty}^{t} x(\tau)d\tau$$

$$y[n]=\sum_{k=-\infty}^{n} x[k]=\sum_{k=-\infty}^{n-1} x[k]+x[n]$$

$$y[n]=y[n-1]+x[n]$$



Ex: contorul de apa/curent etc.

Sisteme cu memorie sau dinamice:

- Diferentierea si integrarea (in timp continuu)
- Diferenta finita si acumulatorul in timp discret

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2.8. Exemple

i) Analizam **liniaritatea si invarianta in timp** pentru un sistem analogic descris prin ecuatia diferentiala cu coeficienti variabili in timp

$$\frac{d^2y(t)}{dt^2}+2t\frac{dy(t)}{dt}+t^2y(t)=x(t)$$

a) Sistem liniar = aditiv + omogen

Sistem aditiv ?

$$x_1(t)\longrightarrow y_1(t)\Rightarrow\frac{d^2y_1(t)}{dt^2}+2t\frac{dy_1(t)}{dt}+t^2y_1(t)=x_1(t)$$

$$x_2(t)\longrightarrow y_2(t)\Rightarrow\frac{d^2y_2(t)}{dt^2}+2t\frac{dy_2(t)}{dt}+t^2y_2(t)=x_2(t)$$

$$\frac{d^2}{dt^2}\left[y_1(t)+y_2(t)\right]+2t\frac{d}{dt}\left[y_1(t)+y_2(t)\right]+t^2\left[y_1(t)+y_2(t)\right]=x_1(t)+x_2(t)$$

$$\Rightarrow x_1(t)+x_2(t)\longrightarrow y_1(t)+y_2(t)$$

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Sistem omogen ?

$$x(t) \longrightarrow y(t) \Rightarrow \frac{d^2 y(t)}{dt^2} + 2t\frac{dy(t)}{dt} + t^2 y(t) = x(t)$$

$$\frac{d^2}{dt^2}\left[ay(t)\right] + 2t\frac{d}{dt}\left[ay(t)\right] + t^2\left[ay(t)\right] = ax(t) \Rightarrow ax(t) \longrightarrow ay(t)$$

b) Invarianta la deplasare in timp

$$x(t) \longrightarrow y(t) \qquad \frac{d^2 y(t)}{dt^2} + 2t\frac{dy(t)}{dt} + t^2 y(t) = x(t)$$

$$x(t-t_0) \longrightarrow y_3(t) \qquad \frac{d^2 y_3(t)}{dt^2} + 2(t-t_0)\frac{dy_3(t)}{dt} + (t-t_0)^2 y_3(t) = x(t-t_0)$$

Daca sistemul ar fi invariant la deplasarea in timp:

$$\frac{d^2 y(t-t_0)}{dt^2} + 2t\frac{dy(t-t_0)}{dt} + t^2 y(t-t_0) = x(t-t_0)$$

Dar $y_3(t) \neq y(t-t_0) \Rightarrow$ sistemul este liniar dar nu si invariant la deplasarea in timp

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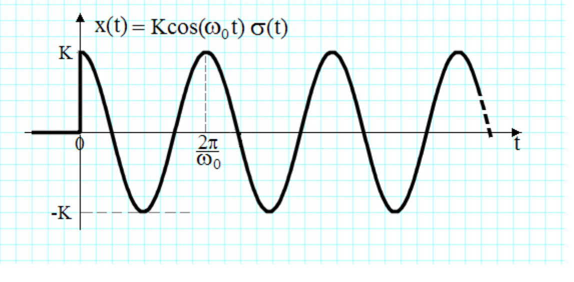
ii) Analizam importanta **conditiilor initiale nule** asupra liniaritatii unui sistem analogic.

Sistem dinamic descris prin ecuatia diferentiala de ordinul unu, cu coeficienti constanti

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

cu semnalul de intrare:

$$x(t) = K\cos\omega_0 t\sigma(t) = \begin{cases} K\cos\omega_0 t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



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Solutia fortata de Kcos(ω_0t), de regim stationar

$$\frac{dy_f(t)}{dt} + 2y_f(t) = K\cos\omega_0 t, \quad t \ge 0$$

$$y_f(t) = \frac{K}{\sqrt{4+\omega_0^2}}\cos(\omega_0 t - \theta), \quad t \ge 0$$

Solutia de regim liber, tranzitorie. Este solutia ecuatiei omogene

$$\frac{dy_{tr}(t)}{dt} + 2y_{tr}(t) = 0 \quad t \in \mathbb{R}$$

$$y_{tr}(t) = Be^{-2t}, \quad t \ge 0 \quad \text{and} \quad y_{tr}(t) = Ce^{-2t}, \quad t < 0$$

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Solutia finala trebuie sa fie continua in t=0

$$y(t) = \begin{cases} y_{tr}(t) + y_f(t), & t \ge 0 \\ y_{tr}(t), & t < 0 \end{cases}$$

$$y(t) = \begin{cases} y_0 e^{-2t} + \frac{K}{\sqrt{4+\omega_0^2}}\left[\cos(\omega_0 t - \theta) - e^{-2t}\cos\theta\right], & t \ge 0 \\ y_0 e^{-2t}, & t < 0 \end{cases}$$

$$y(t) = y_0 e^{-2t} + \frac{K}{\sqrt{4+\omega_0^2}}\left[\cos(\omega_0 t - \theta) - e^{-2t}\cos\theta\right]\sigma(t) \quad t \in \mathbb{R}$$

Pentru $K = 0 : x(t) = 0 \xrightarrow{\text{sistem liniar}} y(t)\big|_{K=0} = y_0 e^{-2t} \equiv 0$

$\Rightarrow$ Numai cu conditie initiala nula, $y_0 = 0$, sistemul este liniar

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iii) Importanta conditiilor initiale nule asupra liniaritatii unui sistem digital

Sistem dinamic discret, descris prin ecuatia cu diferente finite, de ordinul unu, cu coeficienti constanti, cu semnalul de intrare:

$$y[n]-0,5y[n-1]=x[n]\Rightarrow x[n]=K\,cos\,\Omega_0 n\sigma[n]=\begin{cases}K\,cos\,\Omega_0 n\,, & n\ge 0\\ 0\,, & n<0\end{cases}$$

Solutia de regim stationar

$$y_f[n]-0.5y_f[n-1]=K\cos\Omega_0 n=\mathrm{Re}\left\{Ke^{j\Omega_0 n}\right\},\quad n\ge 0$$

$$y_f[n]=A\cos(\Omega_0 n-\theta)$$

$$A=\frac{K}{\sqrt{1.25-\cos\Omega_0}}e^{-j\theta};\quad \theta=\mathrm{arctg}\frac{0.5\sin\Omega_0}{1-0.5\cos\Omega_0}$$

Solutia de regim tranzitoriu

$$y_{tr}[n]-0,5y_{tr}[n-1]=0\ ,\ n\in\mathbb{N}$$

$$y_{tr}[n]=B(0,5)^n\ ,\ n\ge 0\ \text{ si }\ y_{tr}[n]=C(0,5)^n\ ,\ n<0$$

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Solutia finala

$$y[n]=\begin{cases}B(0.5)^n+\dfrac{K}{\sqrt{1.25-\cos\Omega_0}}\cos(\Omega_0 n-\theta), & n\ge 0\\ C(0.5)^n, & n<0\end{cases}$$

Sistem liniar $\Leftrightarrow$ conditii initiale nule: $y[-1]=0$

$$y[n]=\frac{K}{\sqrt{1.25-\cos\Omega_0}}\cos(\Omega_0 n-\theta)\sigma[n]$$

Pentru un sistem de ordin N, conditiile initiale sunt:

$$y[-1]=y[-2]=...=y[-N]=0$$

Pentru un sistem <u>liniar si invariant in timp</u> (SLIT), conditiile initiale sunt nule.

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