2. CONVOLUTION

1

Convolution sum. Response of d.t. LTI systems at a certain input signal

Any signal multiplied by the unit impulse

= the unit impulse weighted by the value of the signal in 0:

$$x[n]\cdot\delta[n]=x[0]\cdot\delta[n]$$

Delayed unit impulse: $x[n]\cdot\delta[n-k]=x[k]\cdot\delta[n-k]$

Any signal can be "broken" into a sum of shifted and weighted unit impulses

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\cdot\delta[n-k]$$

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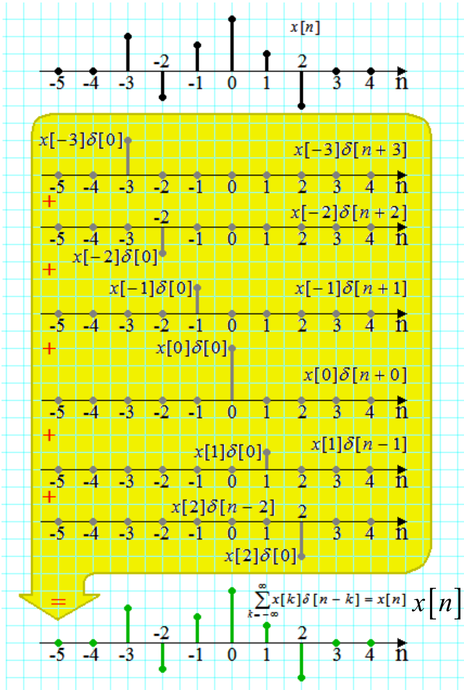
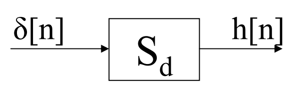
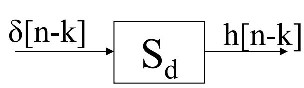


$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k]$$

- The impulse response: response to δ[n]



LTI system

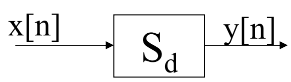


LTI system

Convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] =$$

$$= x[n] * h[n] = h[n] * x[n]$$



LTI system

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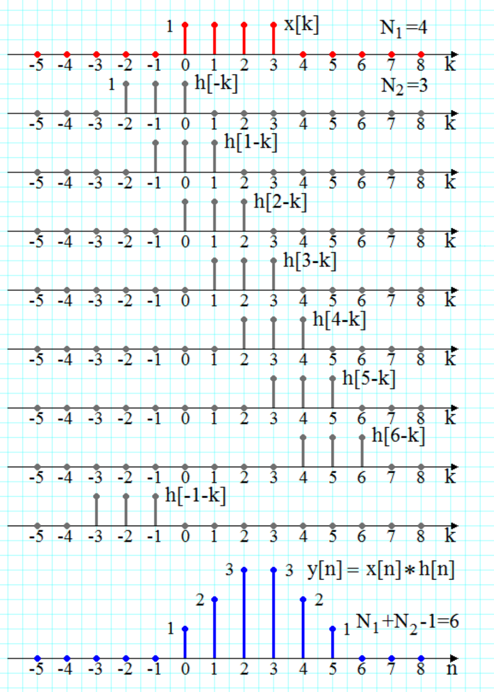

Example 1) The convolution has finite support

$$(x * y)[n] = \sum_{k=-\infty}^{\infty} x[k] y[n-k]$$

two signals with finite durations N_1 and N_2

Convolution - duration of $N_1 + N_2 - 1$.

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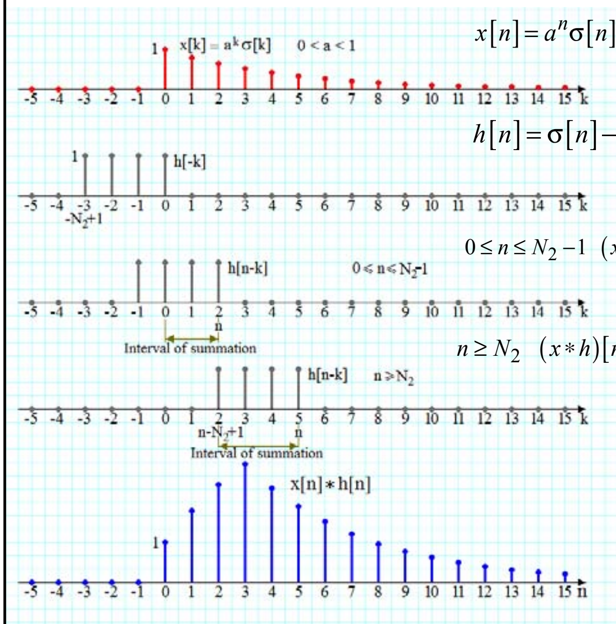
Example 2) The convolution has infinite support



$$x[n]=a^n\sigma[n]$$

$$h[n]=\sigma[n]-\sigma[n-N_2]$$

$$0\le n\le N_2-1 \quad (x*h)[n]=\sum_{k=-\infty}^{\infty} a^k=\frac{1-a^{n+1}}{1-a}$$

$$n\ge N_2 \quad (x*h)[n]=\sum_{k=n-N_2+1}^{n} a^k=a^{n-N_2+1}\frac{1-a^{N_2}}{1-a}$$

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Causal Discrete-Time Signals

If the input signal and the system are causal then the output signal is also causal.

$$x[n]\equiv 0 \text{ and } h[n]\equiv 0\,,\, n<0 \Rightarrow y[n]=\sum_{k=0}^{n} x[k]h[n-k]$$

If the system causal:

$$h[n]\equiv 0\,,\, n<0 \quad \Leftrightarrow \quad h[n]=h[n]\cdot\sigma[n]\,,\,\forall n\in Z$$

$$y[n]=\sum_{k=0}^{\infty} x[n-k]h[k]=\sum_{k=-\infty}^{n} x[k]h[n-k]$$

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BIBO stability

- “bounded input bounded output” (BIBO) stable system
- BIBO stability condition

A discrete–time LTI system is stable if and only if its impulse response is absolutely summable

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty, h[n] \in l^1$$

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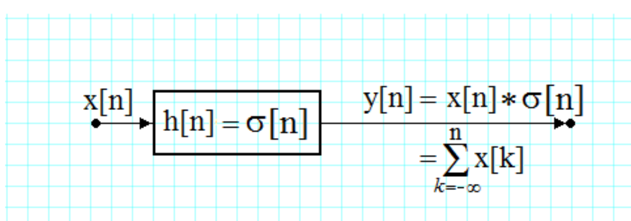
The stability of the accumulator

- h[n] - the unit step σ[n], bounded signal
- finite difference eq. (x[n] causal) :

$$y[n]=x[n]*\delta[n] \Rightarrow y[n]=\sum_{k=0}^{n} x[k]$$



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- The corresponding output signal :

$$y[n] = \sum_{k=0}^{n} 1 = 1 + 1 + \ldots + 1 = n + 1$$

- The output signal is not bounded.
- The accumulator is **unstable**.
- Used in practice,
  - limited interval of summation n , or,
  - from time to time, the output set on zero.

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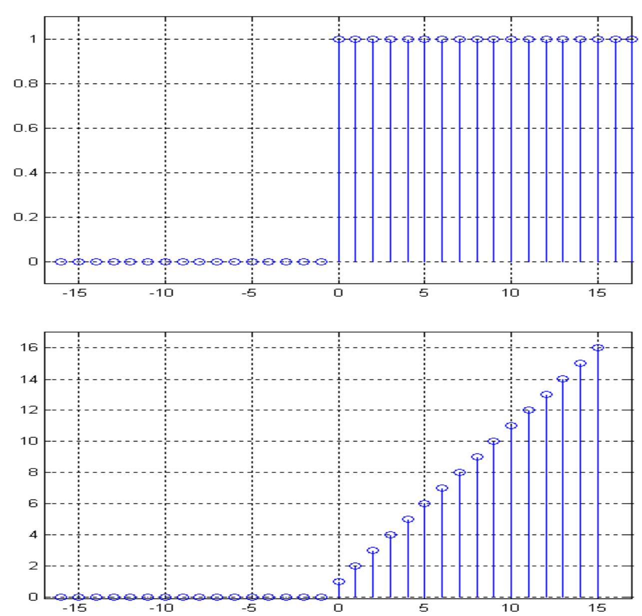
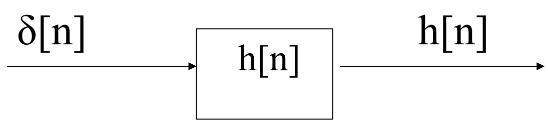


Properties of convolution
Neutral element

- unit impulse δ[n] is **neutral element**.

  x[n]* δ[n] = x[n], for any signal x[n]



h[*n*] - the system's impulse response.

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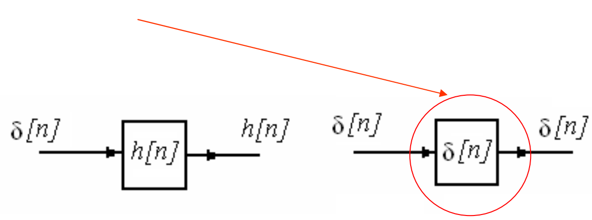
Identity system

- impulse response δ[n]
- input δ[n] → output δ[n].



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Delay system

Impulse respone $h[n]=\delta[n-n_0]$.

Response: a time shifted variant of the input signal

$$x[n]*\delta[n-n_0]=x[n-n_0]$$

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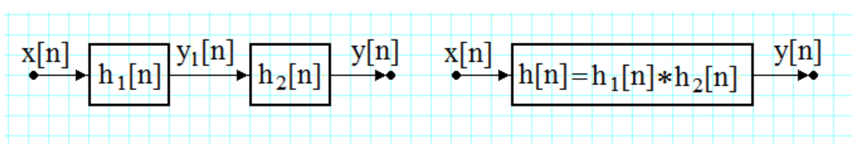
Associativity. Series interconnection of 2 d.t. LTI systems

- two LTI systems $h_1[n]$, $h_2[n]$ connected in series



$$y_1[n]=x[n]*h_1[n]$$

$$y[n]=y_1[n]*h_2[n]=(x*h_1)[n]*h_2[n]$$

$$y[n]=x[n]*(h_1*h_2)[n]$$

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Associativity

- The convolution is **associative**.

$$(x * h_1)[n] * h_2[n] = x[n] * (h_1[n] * h_2[n])$$

- The response of the equivalent system at the unit impulse signal

$$h[n] = h_1[n] * h_2[n]$$

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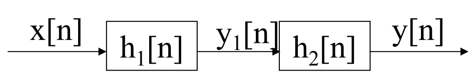
Stability for series interconnection of two 2 d.t. LTI systems

- By the cascade connection of **two stable systems** another **stable system** is obtained.

$$h_1[n] \in l^1,\ h_2[n] \in l^1 \Rightarrow (h_1 * h_2)[n] \in l^1$$



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Commutativity

- The convolution sum is **commutative**.
- The equivalent system is

$$h[n] = h_1[n] * h_2[n] = h_2[n] * h_1[n]$$

- their order is **not important** in the cascade connection!!!

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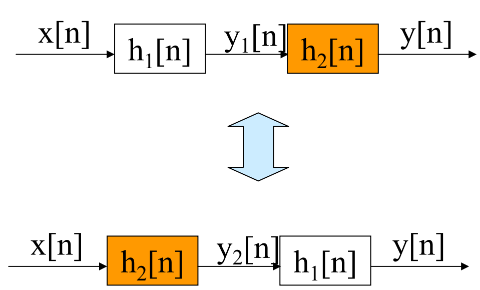
The order in series connection



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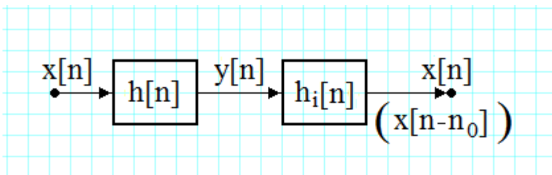
The Inverse System

Two systems connected in series

The output of the second system - the original input signal

$$h[n] * h_i[n] = \delta[n]$$



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Inverse system - example

- delay system:

$$h[n] = \delta[n - n_0]$$

- **inverse system of a delay system**

$$h_1[n] = \delta[n + n_0]$$

- time shift in the other sense of the time axis.
- not a causal system

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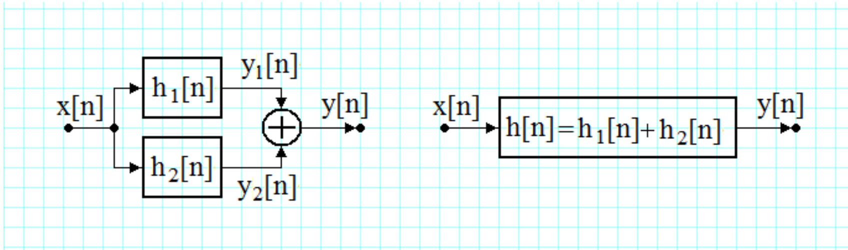
Distributivity.
Parallel interconnection of dt LTI systems



- convolution is distributive with respect to addition

$$(x*h_1)[n]+(x*h_2)[n]=(x*(h_1+h_2))[n]$$

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Proof

$$y[n]=y_1[n]+y_2[n]=(x*h_1)[n]+(x*h_2)[n]=$$

$$=\sum_{k=-\infty}^{\infty}x[k]\cdot h_1[n-k]+\sum_{k=-\infty}^{\infty}x[k]\cdot h_2[n-k]=$$

$$=\sum_{k=-\infty}^{\infty}x[k]\cdot(h_1[n-k]+h_2[n-k])=$$

$$=(x*(h_1+h_2))[n]$$

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Response of a d.t. LTI system at the unit step. The unit step response

- The **unit step response** s[n].

$$x[n]=\sigma[n],$$

$$y[n]=s[n]=h[n]*\sigma[n]=\sum_{k=-\infty}^{n} h[k]$$

$$\boxed{h[n]=s[n]-s[n-1]}$$

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- For causal systems:

$$\boxed{h[n]\equiv 0,\quad \text{for } n<0 \Rightarrow s[n]=\sum_{k=0}^{n} h[k]}$$

- For an accumulator the unit step response a ramp signal (see previous slides).

$$x[n]=\sigma[n]\Rightarrow s[n]=n+1$$

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Finite Impulse (FIR) and Infinite Impulse (IIR) Response d.t. Systems

- There are two types of impulse responses
  - with finite duration (FIR) and
  - with infinite duration (IIR).

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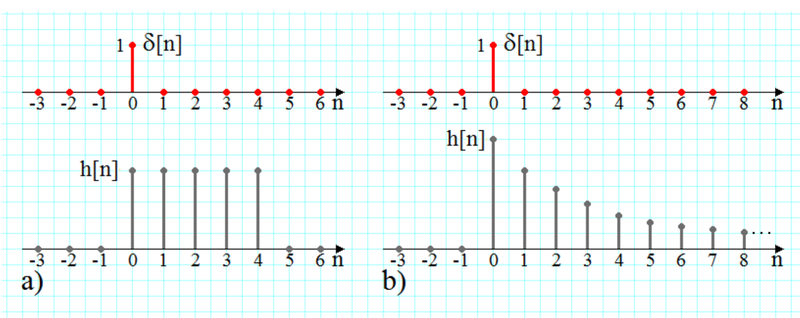
Finite Impulse (FIR) and Infinite Impulse (IIR) Response d.t. Systems



FIR IIR

$$h[n] = \begin{cases} \dfrac{b_n}{a_0}, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

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FIR

- The finite difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- a FIR system has the property:

$$a_0 \neq 0 \text{ and } a_1 = a_2 = \ldots = a_N = 0$$

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FIR

- the output signal using the finite diff.eq.:

$$y[n] = \sum_{k=0}^{M} \frac{b_k}{a_0} x[n-k]$$

- the output signal using convolution:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

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FIR

- Identification :

$$h[k] = \begin{cases} \dfrac{b_k}{a_0}, & k = 0, 1, ..., M \\ 0, & \text{in rest} \end{cases}$$

- The impulse response has **finite duration M+1** – hence the name "**finite impulse response system**"

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IIR systems

- If the last proposition is not verified, for example if

$$a_1 \neq 0 \text{ and } a_0 \neq 0$$

- then we obtain a IIR system.

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IIR systems - example

$$y[n]-0.5y[n-1]=x[n]$$

initial condition

$y[-1]=0$

$$x[n]=\delta[n]\Rightarrow y[n]=h[n]$$

- $n=0 \Rightarrow h[0]=1$
- $n=1 \Rightarrow h[1]=0.5$
- $n=2 \Rightarrow h[1]=0.5^2$
- $n=3 \Rightarrow h[1]=0.5^3$, and so on

$$h[n]=0.5^n\sigma[n]$$

-infinite duration

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Implementation of d.t. LTI systems described by finite differences equations with constant coefficients

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Implementation

- D.t. LTI systems are mathematically described by finite difference equations with constant coefficients.
- Implemented using standard sub-systems:
  - delay systems,
  - multipliers with constant,
  - adders.

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Implementation

- 1^{st} order LTI system

$$a_0 y[n] + a_1 y[n-1] = \underbrace{b_0 x[n] + b_1 x[n-1]}_{z[n]}$$

•a delay sub-system: input signal x[n] into x[n-1]

•two multipliers with constants b_0 and b_1 : $b_0x[n]$ and $b_1x[n-1]$

•one adder

Implemented using the system a)

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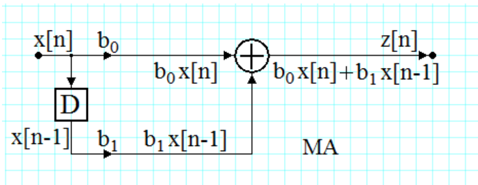
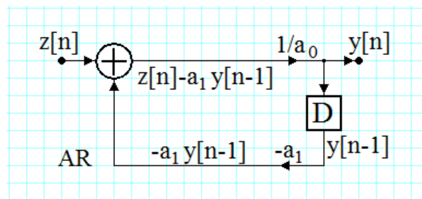
Direct form I



Moving average part

$$y[n]=\frac{1}{a_0}\left(z[n]-a_1 y[n-1]\right)$$



Autoregressive part

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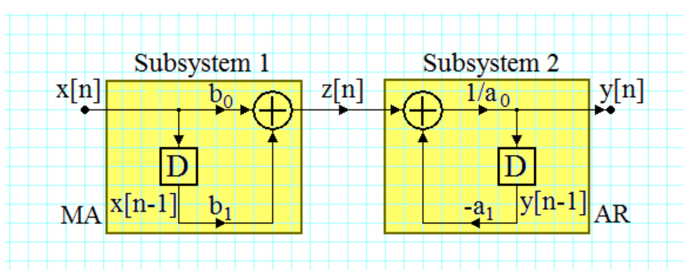
$$a_0 y[n]+a_1 y[n-1]=b_0 x[n]+b_1 x[n-1]$$



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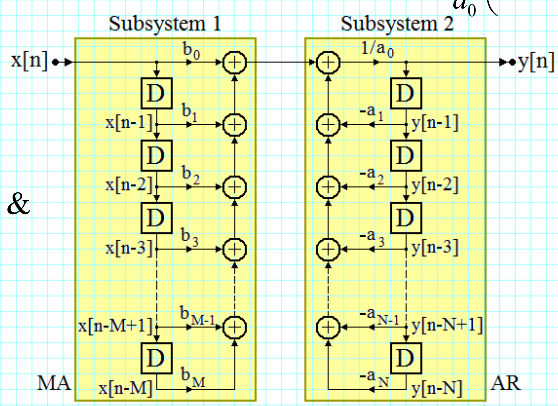
N^{th} order system $\quad \sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$;

$$z[n] = \sum_{k=0}^{M} b_k x[n-k] \qquad y[n] = \frac{1}{a_0}\left(z[n] - \sum_{k=1}^{N} a_k y[n-k] \right)$$

delay systems, multipliers & adders



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- For a FIR discrete-time system :

$$y[n] = \frac{1}{a_0}(z[n])$$

- sub-system 2 = multiplier with constant $1/a_0$

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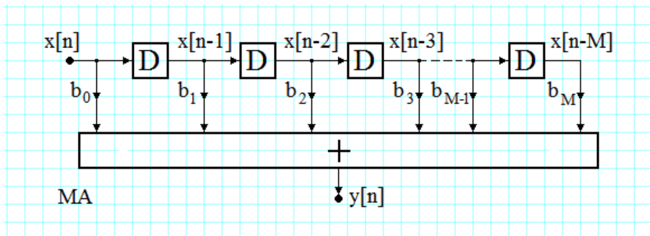
FIR system - transversal form

- substitute the *M* adders from subsystem 1 with a single adder with *M* inputs



- we have supposed that a_0=1.

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Direct form II

- For 1st order LTI system direct form I: <u>series</u> interconnection of two sub-systems, 1 and 2.
- in the series interconnection, the systems' order is not important
- Reverse order and have an equivalent form of the direct form I : series interconnection of the sub-system 2 with the sub-system 1.

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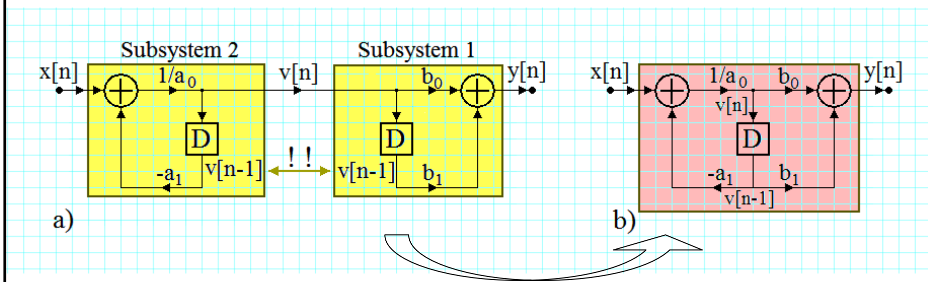
Direct form II

•Systems' order is not important (series interconnection)
•Reverse order - series connection of subsystem 2 with subsystem 1
•Equivalent form of the direct form I : direct form II



Remove redundant delay system

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- two times the same signal v[n-1] -system a).
- So, a **delay subsystem is redundant**.
- Remove the delay subsystem ⇒ implementation b).
- **direct form II** of a 1st order LTI d.t. system.
- **minimum number of subsystems** (delay systems, multipliers and adders).

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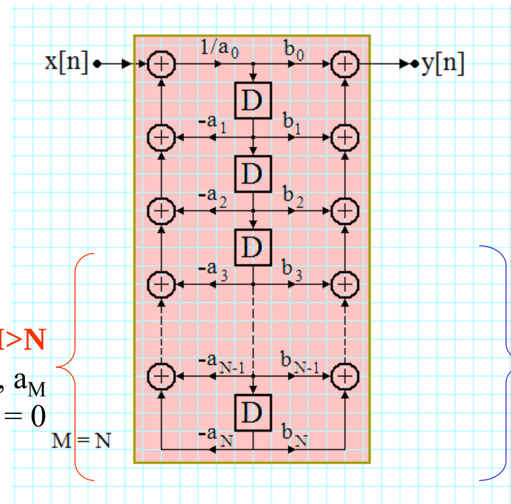
Direct form II for N'th order system



M>N

$a_{N+1}, a_{N+2}, \ldots, a_M = 0$

M<N

$b_{M+1}, b_{N+2}, \ldots, b_N = 0$

- We have supposed that M=N
- **If M>N** then we can suppose that

$$a_{N+1}, a_{N+2}, \ldots, a_M = 0$$

- and we can use the same implementation.
- **If M<N** then we can suppose that

$$b_{M+1}, b_{N+2}, \ldots, b_N = 0$$

- and we can use the same implementation.

The convolution product.

The response of c.t. LTI systems at a specified input signal

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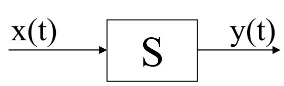
Convolution product. C.t. signals

- C.t. LTI system, mathematically described by the operator *S*.
- find the output signal y(t) when the input signal x(t) is known.



LTI system

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Convolution product. C.t. signals

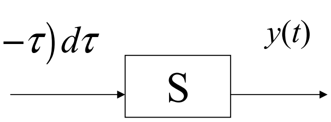
- **Reminder (intro):** Filtering property of the Dirac unit impulse, δ(t)

$$\int_{-\infty}^{\infty} \varphi(\tau)\delta(\tau)\,d\tau = \varphi(0)$$

- Function test x(τ), and as impulse the shifted version with t (time is τ)

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau = x(t)$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$



LTI system

•the output signal

$$y(t) = S\{x(t)\}$$

$$= S\left\{\int_{-\infty}^{\infty} x(\underbrace{\tau}_{\text{const.}})\,\delta(t-\underbrace{\tau}_{\text{const.}})\,d\tau\right\}$$

$$= \int_{-\infty}^{\infty} x(\tau)\,S\{\delta(t-\tau)\}\,d\tau$$

Convolution product. Definition

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

- **convolution product of continuous-time signals** *x* and *h*.
- compute the response of a given system, with known impulse response (*h*) to a known input signal (*x*)

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Neutral element

- **Neutral element** for convolution: Dirac distribution, δ(t).

$$x(t) * \delta(t) = x(t), \quad \text{for any signal } x(t)$$

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Commutativity

- With the notation: $t - u = \tau$, we have:

$$(f * g)(t) = \int_{-\infty}^{\infty} g(u) f(t-u)\, du = (g * f)(t)$$

- So, the convolution product is **commutative** almost everywhere.

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Some remarks

- If the input signal $x(t) \in L^1$ and $h(t) \in L^1$ (the system is stable)
- Then the convolution exists and the output signal

$$y(t)=x(t)*h(t) \in L^1$$

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Some remarks

- If the input signal $x(t) \in L^2$ and the impulse response system $h(t) \in L^2$
- then the convolution product $x(t)*h(t)$ *exists, is bounded and continuous*

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Some remarks

- If the input signal $x(t) \in L^2$ and the system is stable $h(t) \in L^1$
- Then the output signal has a **finite energy**

$$y(t)=x(t)*h(t) \in L^2$$

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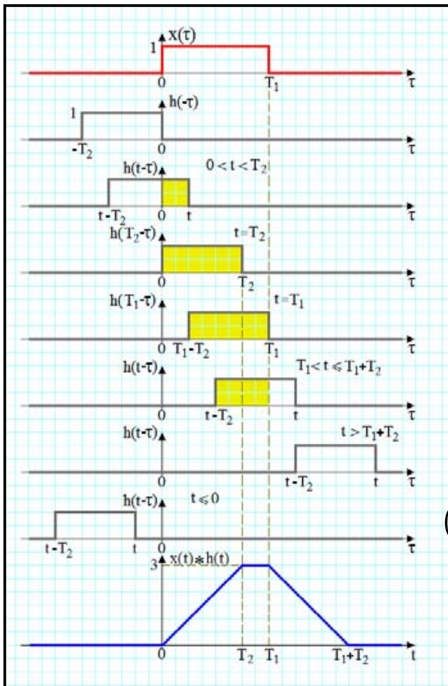

Example 1)

$$f(t) = \sigma(t) - \sigma(t - T_1)$$
$$g(t) = \sigma(t) - \sigma(t - T_2)$$

length of the convolution's support = $T_1 + T_2$ (sum of lengths of the supports of the two functions)

$$(f * g)(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \le t < T_2 \\ T_2, & T_2 \le t \le T_1 \\ T_1 + T_2 - t, & T_1 \le t \le T_1 + T_2 \\ 0, & t > T_1 + T_2 \end{cases}$$

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- Denote g(-τ) = z(τ)
- Time-shifting of z(τ) with t to the right

$$t < 0,\ f(\tau) g(t - \tau) = 0;$$

$$0 < t \le T_2,\ (f * g)(t) = \int_0^t d\tau = t;$$

$$T_2 \le t \le T_1,\ (f * g)(t) = \int_{t - T_2}^{t} d\tau = T_2;$$

$$T_1 \le t \le T_1 + T_2,\ (f * g)(t) = \int_{t - T_1}^{T_2} d\tau = T_1 + T_2 - t;$$

$$t > T_1 + T_2,\ f(\tau) f(t - \tau) = 0,\ (f * g)(t) = 0.$$

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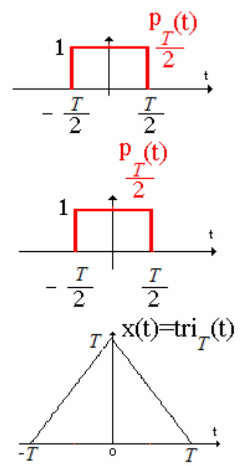
Example 2)



$$f(t)=\sigma\left(t+\frac{T}{2}\right)-\sigma\left(t-\frac{T}{2}\right);$$

$$g(t)=f(t);$$

$$(f*g)(t)=T\left(1-\frac{|t|}{T}\right)\left(\sigma(t+T)-\sigma(t-T)\right);$$

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Example 3)

$$f(t)=\frac{1}{\sqrt{|t|}}\cdot\frac{1}{|t|+1},\quad f(t)*f(t)=?$$

- $f(0)\rightarrow\infty$; $f(t)$ *and* $|f(t)|$ are even.
- function f belongs to L^1

$$\int_{-\infty}^{\infty}|f(t)|dt=2\int_{0}^{\infty}\frac{du}{u^2+1}=2\operatorname{arctg}u\Big|_0^{\infty}=2\cdot\frac{\pi}{2}=\pi<\infty$$

- $f*f$ is convergent a.e.; but not for $t=0$

$$(f*f)(0)=\int_{-\infty}^{\infty}f(\tau)\cdot f(-\tau)d\tau=\int_{-\infty}^{\infty}\frac{1}{\tau}\cdot\frac{1}{(|\tau|+1)^2}d\tau$$

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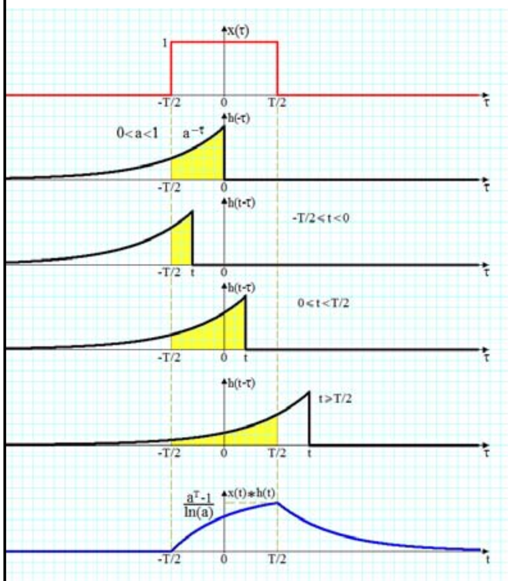

Example 4)

$$f(t)=\sigma\left(t+\frac{T}{2}\right)-\sigma\left(t-\frac{T}{2}\right);$$

$$g(t)=a^{t}\sigma(t)\ ,\ 0<a<1;$$

$(f*g)\in L^{2}$ has infinite support.

$$(f*g)(t)=\frac{1}{\ln\left(\frac{1}{a}\right)}\left[\left(1-a^{t+\frac{T}{2}}\right)\sigma\left(t+\frac{T}{2}\right)-\left(1-a^{t-\frac{T}{2}}\right)\sigma\left(t-\frac{T}{2}\right)\right].$$

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- $t < -T/2$, $f*g(t) = 0$.
- $t >$ -T/2 partial superposition - for $-T/2 < t < T/2$:

$$(f*g)(t)=\int_{-\frac{T}{2}}^{t} a^{t-\tau}d\tau=\frac{1}{\ln\frac{1}{a}}\left(-a^{t+\frac{T}{2}}+1\right)$$

•$t >$ -T/2 complete superposition - for $t \geq T/2$:

$$(f*g)(t)=\int_{-\frac{T}{2}}^{\frac{T}{2}} a^{t-\tau}d\tau=\frac{1}{\ln\frac{1}{a}}\left(a^{t-\frac{T}{2}}-a^{t+\frac{T}{2}}\right).$$

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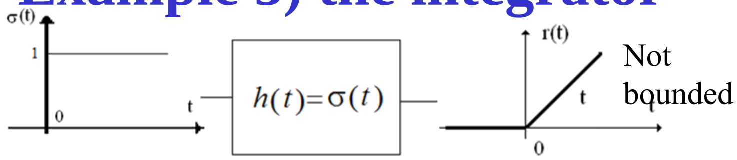
Example 5) the integrator



- impulse response = σ(*t*).

$$x(t)=\delta(t)\Rightarrow y(t)=h(t)=\int_{-\infty}^{t}\delta(\tau)d\tau=\sigma(t)$$

- step response = ramp signal:

$$(\sigma*\sigma)(t)=\int_{-\infty}^{\infty}\sigma(\tau)\sigma(t-\tau)\,d\tau=\begin{Bmatrix} t, & t\geq 0 \\ 0, & t<0 \end{Bmatrix}=t\cdot\sigma(t).$$

- The result is not bounded, so the integrator is not stable.
- Despite this fact, the integrators can be used in practice, but only for finite duration input signals (case in which the output is bounded).

Associativity

- The convolution is associative
  $$\left(f(t)*g(t)\right)*h(t)=f(t)*\left(g(t)*h(t)\right)$$
- If

1. f, g, h $\in L^1$;
2. f, g, h $\in L^1_{loc}$ and two of them have compact support;
3. f, g, h $\in L^1_{loc}$ and all three have like support a closed set included in $[0,\infty)$.

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Unit step response of an LTI system

- response to the unit step signal.

$$s(t)=h(t)*\sigma(t)=\int_{-\infty}^{t}h(\tau)d\tau$$

- Its derivative is the impulse response.

$$s'(t)=h(t)$$

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- response of the system *h* to the ramp signal

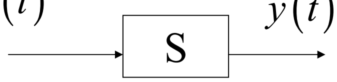
$$x(t)=t\sigma(t) \;\Rightarrow\; y(t)=S\{t\sigma(t)\}=h*(t\sigma(t));$$

- The 2nd derivative of y(t) is h(t).

$$y''(t)=h*(t\sigma(t))''=h*\delta=h.$$

$x(t)=t\sigma(t)$

$y(t)=h(t)*t\sigma(t)$



LTI system

$y''(t)=h(t)$

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- find the impulse response h(t) of a system
  - **the derivation of its step response or**
  - **the double derivation of its response to a ramp signal**.

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The unit step response of a causal LTI system

- for a causal system, impulse response = zero for $t<0$ or $h(t) = h(t)\cdot\sigma(t)$

$$s(t) = h(t) * \sigma(t) = \int_0^t h(\tau)d\tau$$

- Causal input signal and system $\Rightarrow$ causal output

$$y(t) = \int_0^t x(t-\tau)h(\tau)d\tau$$

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BIBO stability for continuous-time LTI systems

- "bounded input bounded output" (BIBO) stable system
- BIBO stability condition

A continuous–time LTI system is stable <u>if and only if</u> its impulse response is absolutely integrable

$$\int_{-\infty}^{\infty} |h(\tau)|d\tau < \infty, h(t) \in L^1$$

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An example – the integrator

$h(t) = \sigma(t) \notin L^1;\ \ y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

$y(t) = \int_{0}^{t} x(\tau)d\tau$ (input signal is causal)

Bounded input $x(t) = \sigma(t) \Rightarrow$ unbounded output $y(t) = \int_{0}^{t} 1 \cdot d\tau = t.$

If $x(t)$ has finite duration then $y(t)$ is bounded.

•the integrator is not stable.
•It can be used in practice because all the practical signals have finite duration.

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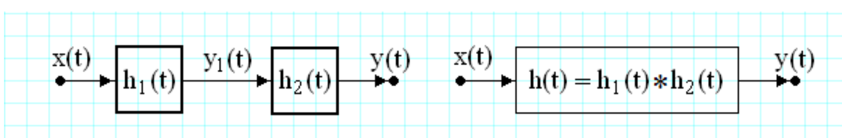
Practical significance of the convolution product's properties



•Equivalent system for series interconnection of 2 systems has impulse response

$$h(t) = h_2(t) * h_1(t) = h_1(t) * h_2(t)$$

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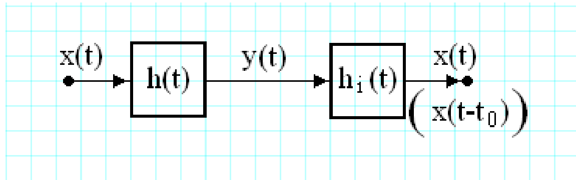
Inverse system. Identity system



- Two systems connected in series
- Output – original input signal

$$h(t) * h_i(t) = \delta(t)$$

- The system connected in series with h(t) and $h_i(t)$ is an identity system

$$y(t)=x(t)$$

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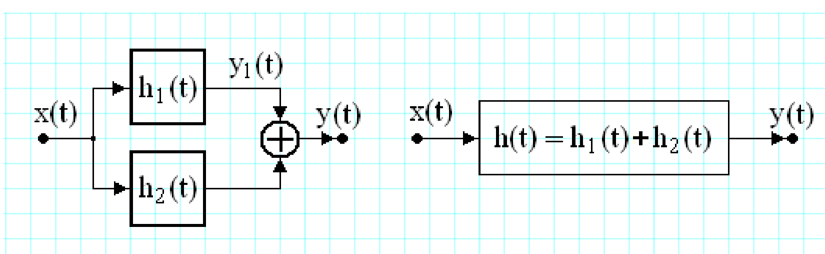
Parallel interconnection of LTI systems



•Equivalent system for parallel interconnection of 2 systems has impulse response

$$h(t) = h_1(t) + h_2(t)$$

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Implementation of c.t. LTI systems with linear differential equations & constant coefficients

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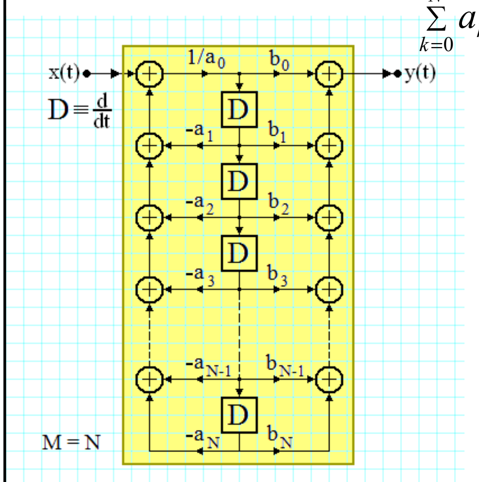
Direct form II with differentiators.

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{N} b_k \frac{d^k x(t)}{dt^k} \quad a_N \neq 0$$



- •direct form II: delay systems (discrete-time) → differentiators (cont. time).
- •difficult to construct differentiators in continuous-time

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Direct form II with integrators.

- it is preferable to use integrators.
- Integrate N times a differential equation ⇒ an integral equation. With the notations

$$y_{(0)}, y_{(1)}, \ldots, y_{(N)}, \ x_{(0)}, x_{(1)}, \ldots, x_{(N)}$$

- we obtain the integral equation :

$$\sum_{k=0}^{N} a_k y_{(N-k)}(t) = \sum_{k=0}^{N} b_k x_{(N-k)}(t).$$

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$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{N} b_k \frac{d^k x(t)}{dt^k} \quad a_N \neq 0$$

$$y_{(0)}(t) = y(t),$$

$$y_{(1)}(t) = y(t) * \sigma(t) = \int_{-\infty}^{t} y(\tau_1) d\tau_1$$

$$y_{(2)}(t) = y(t) * \sigma(t) * \sigma(t) = \int_{-\infty}^{t} \int_{-\infty}^{\tau_2} y(\tau_1) d\tau_1 d\tau_2$$

....

$$y_{(k)}(t) = y_{(k-1)}(t) * \sigma(t) = \int_{-\infty}^{t} \int_{-\infty}^{\tau_k} \int_{-\infty}^{\tau_{k-1}} \ldots \int_{-\infty}^{\tau_2} y(\tau_1) d\tau_1 d\tau_2 \ldots d\tau_{k-1} d\tau_k$$

$$x_{(0)}(t) = x(t) \ ; x_{(1)}(t) = x(t) * \sigma(t), \ldots$$

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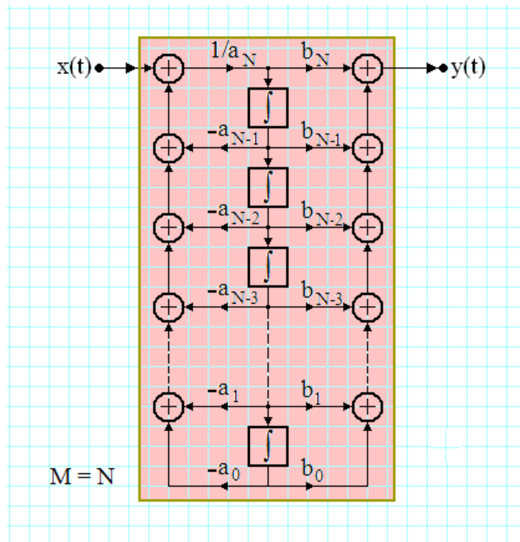
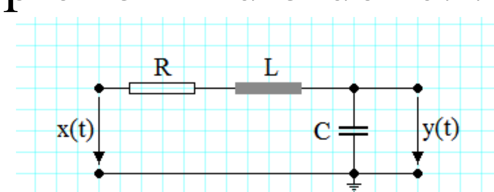


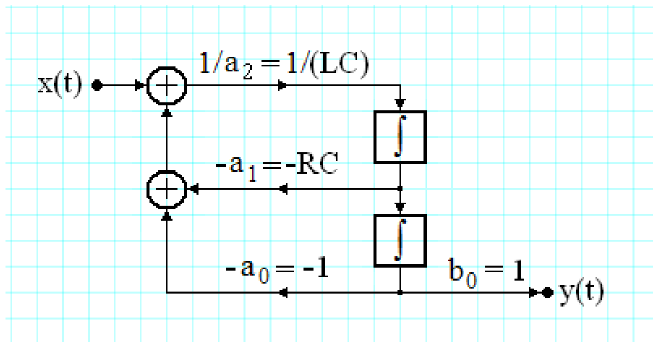
Example for 2nd order c.t. system



$$LC\frac{d^2y(t)}{dt^2}+RC\frac{dy(t)}{dt}+y(t)=x(t)$$

$$\sum_{k=0}^{N}a_k y_{(n-k)}(t)=\sum_{k=0}^{N}b_k x_{(n-k)}(t),$$

$$LCy_{(0)}(t)+RCy_{(1)}(t)+y_{(2)}(t)=x_{(2)}(t)$$



- identification :

$a_0 = 1$; $a_1 = RC$; $a_2 = LC$ $\quad$ $b_0 = 1$

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- Using these coefficients we can write the corresponding integral equation
- we obtain direct form II with integrators.

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Transversal structure for FIR systems

- continuous-time FIR systems have the impulse response :

$$h(t) = \sum_{k=0}^{N} h_k \delta(t - kT).$$

- implemented using the transversal structure

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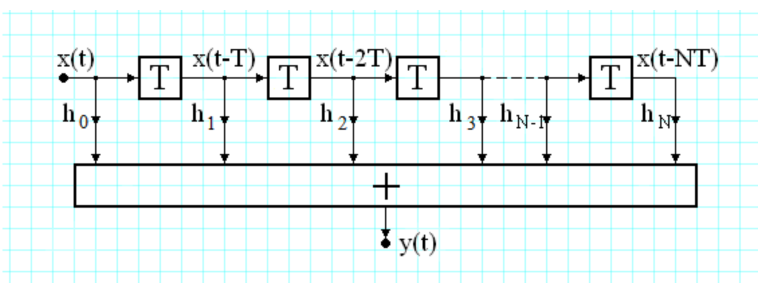

$$y(t) = x(t)h_0 + x(t-T)h_1 + \ldots + x(t-NT)h_N,$$

$$y(t) = x(t) * \sum_{k=0}^{N} h_k \delta(t-kT) = x(t) * h(t),$$

$$h(t) = \sum_{k=0}^{N} h_k \delta(t-kT).$$

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