

Tabelul 5.1 Proprietățile seriei Fourier în timp discret

Semnalul discret periodic	Coefficienții seriei Fourier exponențiale
$x[n] , x[n+N] = x[n] = x[(n)_N]$	$\{c_k^x\} ; c_{k+N}^x = c_k^x = c_{(k)_N}^x$
$y[n] , y[n+N] = y[n] = y[(n)_N]$	$\{c_k^y\} ; c_{k+N}^y = c_k^y = c_{(k)_N}^y$
$ax[n] + by[n]$	$\{ac_k^x + bc_k^y\}$
$x[n-n_o] , n_o \in \mathbb{R}$	$\{e^{-jk\frac{2\pi}{N}n_o} c_k^x\}$
$x^*[n]$	$\{(c_k^x)^*\}$
$x[-n]$	$\{c_{-k}\}$
$x_{(m)}[n] = \begin{cases} x[n/m] , & n \div m \\ 0 , & \text{în rest} \end{cases}$ <i>perioada mN</i>	$\{\frac{1}{m}c_k^x\} ; \text{perioada } mN$
$x[n] \otimes y[n] = \sum_{k \in \langle N \rangle} x[k]y[n-k]$	$\{Nc_k^x c_k^y\}$
$\frac{1}{N_o} \sum_{n=0}^{N_o-1} x^*[n]y[n+k]$	$\{(c_k^k)^* \cdot c_k^y\}$
$x[n] \cdot y[n]$	$\{c_k^x \otimes c_k^y\} = \left\{ \sum_{m \in \langle N \rangle} c_{k-m}^x c_m^y \right\}$
$x[n] - x[n-1]$	$\left\{ \left(1 - e^{jk\frac{2\pi}{N}}\right) c_k^x \right\}$
$\sum_{k=-\infty}^n x[k] ; c_o^x = 0$	$\left\{ c_k^x / \left(1 - e^{-jk\frac{2\pi}{N}}\right) \right\} , c_o^x = 0$
$x[n] \in \mathbb{R}$	$c_k = (c_{-k})^*$ $ c_k = c_{-k} ; \text{Arg } c_k = -\text{Arg } c_{-k}$ $\text{Re}\{c_k\} = \text{Re}\{c_{-k}\} ; \text{Im}\{c_k\} = -\text{Im}\{c_{-k}\}$
$x_p[n] , x[n] \in \mathbb{R}$	$\{\text{Re}\{c_k^x\}\}$
$x_i[n] , x[n] \in \mathbb{R}$	$\{j\text{Im}\{c_k^x\}\}$
$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) ^2 d\Omega$	

Tabelul 5.2 Proprietățile transformării Fourier în timp discret

Semnalul discret aperiodic	Transformata Fourier
$x[n]$	$X(\Omega) ; X(\Omega + 2\pi) = X(\Omega)$
$y[n]$	$Y(\Omega) ; Y(\Omega + 2\pi) = Y(\Omega)$
$ax[n] + by[n]$	$aX(\Omega) + bY(\Omega)$
$x[n - n_o]$	$e^{-j\Omega n_o} X(\Omega)$
$e^{j\Omega_o n} x[n]$	$X(\Omega - \Omega_o)$
$x^*[n]$	$X^*(-\Omega)$
$x[-n]$	$X(-\Omega)$
$x_{(k)}[n] = \begin{cases} x[n/k] , & n : k \\ 0 , & \text{în rest} \end{cases}$	$X(k\Omega)$
$x[n] * y[n]$	$X(\Omega) Y(\Omega)$
$\check{x}^*[k] * y[k]$	$X^*(\Omega) Y(\Omega)$
$x[n] y[n]$	$\frac{1}{2\pi} X(\Omega) \otimes Y(\Omega) = \frac{1}{2\pi} \int_{2\pi} X(u) Y(\Omega - u) du$
$x[n] - x[n - 1]$	$(1 - e^{-j\Omega}) X(\Omega)$
$\sum_{k=-\infty}^n x[k]$	$\frac{X(\Omega)}{1 - e^{-j\Omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - k 2\pi)$
$nx[n]$	$j \frac{d}{d\Omega} X(\Omega)$
$x[n] \in \mathbb{R}$	$X(\Omega) = X^*(-\Omega)$ $ X(\Omega) = X(-\Omega) ; \text{Arg} X(\Omega) = -\text{Arg} X(-\Omega)$ $\text{Re}\{X(\Omega)\} = \text{Re}\{X(-\Omega)\} ; \text{Im}\{X(\Omega)\} = -\text{Im}\{X(-\Omega)\}$
$x_p[n] , x[n] \in \mathbb{R}$	$\text{Re}\{X(\Omega)\}$
$x_i[n] , x[n] \in \mathbb{R}$	$j \text{Im}\{X(\Omega)\}$
$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{2\pi} X(\Omega) ^2 d\Omega$	

Tabelul 5.3. Perechi semnal-transformată Fourier în timp discret

Semnalul	Transformata Fourier	Coefficienții seriei Fourier exponențiale (pentru) semnale periodice
$\sum_{k \in \langle N \rangle} c_k e^{jk \frac{2\pi}{N} n}$	$\sum_{k=-\infty}^{\infty} 2\pi c_k \delta(\Omega - k \frac{2\pi}{N}); \quad c_k = c_{(k)_N}$	$\{c_k\}$
$e^{j\Omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - \Omega_0 - k2\pi)$	$\frac{\Omega_0}{2\pi} = \frac{m}{N} \in Q: \{c_k\}; \quad c_k = \begin{cases} 1, & (k)_N = m \\ 0, & \text{rest} \end{cases}$ $\frac{\Omega_0}{2\pi} \notin Q; \quad \text{semnalul nu este periodic}$
$\cos \Omega_0 n$	$\sum_{k=-\infty}^{\infty} \pi [\delta(\Omega - \Omega_0 - k2\pi) + \delta(\Omega + \Omega_0 - k2\pi)]$	$\frac{\Omega_0}{2\pi} = \frac{m}{N} \in Q: \{c_k\}; \quad c_k = c_{-k} = \begin{cases} 1/2, & (k)_N = m \\ 0, & \text{rest} \end{cases}$ $\frac{\Omega_0}{2\pi} \notin Q; \quad \text{semnalul nu este periodic}$
$\sin \Omega_0 n$	$\sum_{k=-\infty}^{\infty} \frac{\pi}{j} [\delta(\Omega - \Omega_0 - k2\pi) + \delta(\Omega + \Omega_0 - k2\pi)]$	$\frac{\Omega_0}{2\pi} = \frac{m}{N} \in Q: \{c_k\}; \quad c_k = \begin{cases} 1/(2j), & k = m \pm pN \\ -1/(2j), & k = -m \pm pN \\ 0, & \text{rest} \end{cases}$ $\frac{\Omega_0}{2\pi} \notin Q; \quad \text{semnalul nu este periodic}$
$x[n]=1$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - k2\pi)$	$c_k = \begin{cases} 1, & (k)_N = 0 \\ 0, & \text{rest} \end{cases}$
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 \leq n \leq N/2 \end{cases}$ $x[n+N] = x[n]$	$\sum_{k=-\infty}^{\infty} 2\pi \frac{\sin(\frac{2N_1+1}{2} \frac{2k\pi}{N})}{N \sin \frac{2k\pi}{N}} \delta(\Omega - k \frac{2\pi}{N})$	$c_k = \frac{\sin(\frac{2N_1+1}{2} \frac{2k\pi}{N})}{N \sin \frac{2k\pi}{N}} \quad (k)_N \neq 0; \quad c_k = \frac{2N_1+1}{N}; \quad (k)_N = 0$

Semnalul	Transformata Fourier	Coefficienții seriei Fourier exponențiale (pentru) semnale periodice
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\Omega - k \frac{2\pi}{N})$	$c_k = \frac{1}{N}$
$a^n \sigma[n]; \quad a < 1$	$\frac{1}{1 - ae^{-j\Omega}}$	-
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin[(2N_1 + 1) / 2] \Omega}{\sin(\Omega / 2)}$	-
$\frac{\sin \Omega_0 n}{\pi}; \quad 0 < \Omega_0 < \pi$	$X(\Omega) = \begin{cases} 1, & \Omega < \Omega_0 \\ 0, & \Omega_0 < \Omega \leq \pi \end{cases}; X(\Omega + 2\pi) = X(\Omega)$	-
$\delta[n]$	1 (constanta)	-
$\sigma[n]$	$\frac{1}{1 - e^{-j\Omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$	-
$\delta[n - n_0]$	$e^{-j\Omega_0 n}$	-
$(n+1)a^n \sigma[n]; \quad a < 1$	$\frac{1}{(1 - ae^{-j\Omega})^2}$	-
$\frac{(n+m-1)!}{n!(m-1)!} a^n \sigma[n]; \quad a < 1$	$\frac{1}{(1 - ae^{-j\Omega})^m}$	-