Filtrarea semnalelor

http://shannon.etc.upt.ro/teaching/ps/Cap8.pdf

- Filtrare - Modificarea relativa a amplitudinilor componentelor armonice ale unui semnal periodic sau chiar eliminarea sau selectarea anumitor componente armonice;
- Modificarea densitatii spectrale a unui semnal aperiodic, in sensul favorizarii sau defavorizarii unor segmente spectrale.

1

- Inginerul român Augustin Maior a observat ca pe un **circuit telefonic** s-ar putea transmite mai multe convorbiri simultan.
- Pentru aceasta a transferat spectrul vocal al unei convorbiri într-o banda de frecventa, distincta de al altei convorbiri, prin **modulare**.
- La **receptie**, separarea convorbirilor se realizeaza printr-o operatie de **atenuare** a tuturor componentelor spectrale cu exceptia cate unei benzi, in care a fost plasata convorbirea.
- In acest mod, **prin filtrare**, se separa convorbirile ce au fost amestecate.

2

- Filtrarea se realizeaza cu ajutorul unor sisteme liniare si invariante in timp continuu sau discret.
- Filtrele sunt sisteme de convolutie continue sau discrete.

3

Filtrarea ideala

- Filtrele ideale nu pot fi realizate deoarece sunt necauzale.
- Nu este respectata teorema Paley-Wiener:

$$I = \int_{-\infty}^{\infty} \frac{\left|log\left|X\left(\omega\right)\right|\right|}{1+\omega^2} d\omega \text{ nu converge}$$

$$\text{pentru } |\omega| > \omega_c, \quad \left|log\left|X\left(\omega\right)\right|\right| \to \infty$$

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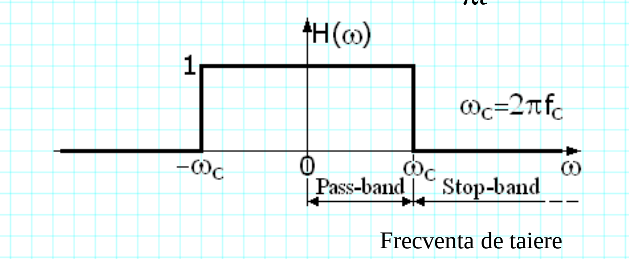
Tipuri de filtre ideale

- Filtrul trece jos FTJ ideal

$$H(\omega) = p_{\omega_c}(\omega) \leftrightarrow h(t) = \frac{\sin \omega_c t}{\pi t}$$

Necauzal



Atenuare:

$$a = \frac{1}{|H(\omega)|} = \begin{cases} 1 & |\omega| < \omega_c \\ \text{infinita} & |\omega| > \omega_c \end{cases} \qquad a[dB] = 20\log\frac{1}{|H(\omega)|} = -20\log|H(\omega)|$$

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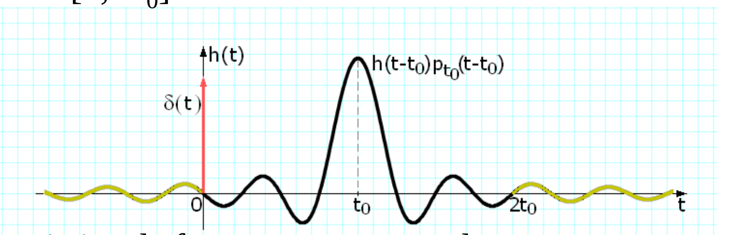
Aproximarea unui filtru ideal printr-un filtru realizabil

- Raspunsul la impuls h(t) se intarzie cu t_0 si se trunchiaza in intervalul $[0, 2t_0]$.



- Caracteristica de frecventa este complexa

$$h(t-t_0) = \frac{\sin \omega_c (t-t_0)}{\pi (t-t_0)} \longleftrightarrow H_{t_0}(\omega) = e^{-j\omega t_0} p_{\omega_c}(\omega)$$

$$|H_{t_0}(\omega)| = |H(\omega)| = p_{\omega_c}(\omega); \quad \Phi_{t_0}(\omega) = -\omega t_0$$

- Modulul este acelasi, dar faza nu ramane zero

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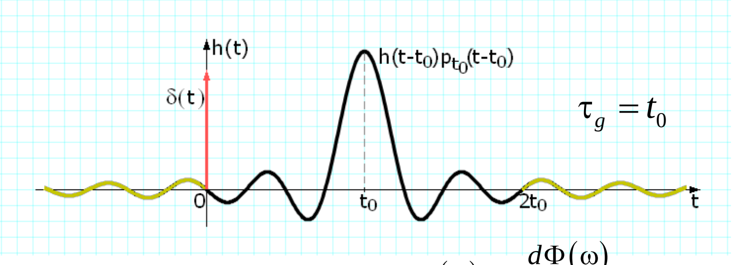

Timpul de intarziere de grup (modulatie) $\tau_g(\omega) = -\frac{d\Phi(\omega)}{d\omega}$

- Raspunsul la impuls $h(t-t_0)p_{t0}(t-t_0)$: cauzal deci sistem realizabil.
- Raspunsul in frecventa este afectat de fenomenul lui Gibbs.
- Cu cat t_0 este mai mare cu atat aproximarea este mai buna
- Se poate folosi in calcule FTJ ideal, ca o limita ce poate fi aproximata oricat de bine in eroare medie patratica.

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- Spectrul semnalului de la iesirea FTJ ideal in functie de spectrul semnalului de intrare

$$Y(\omega) = H(\omega)X(\omega) = \begin{cases} X(\omega), & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

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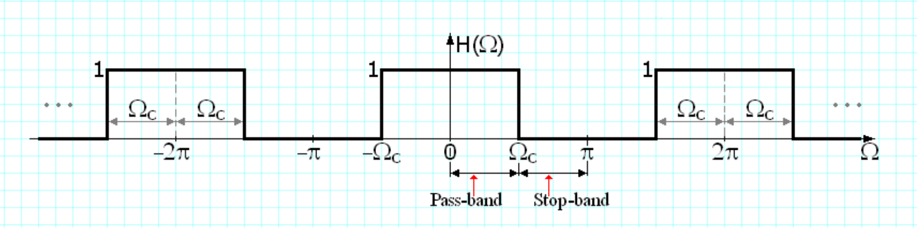
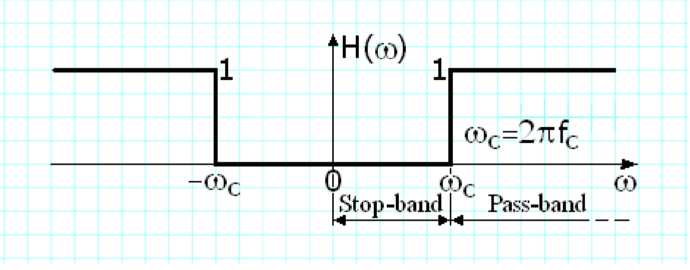
Filtrul trece jos ideal in timp discret



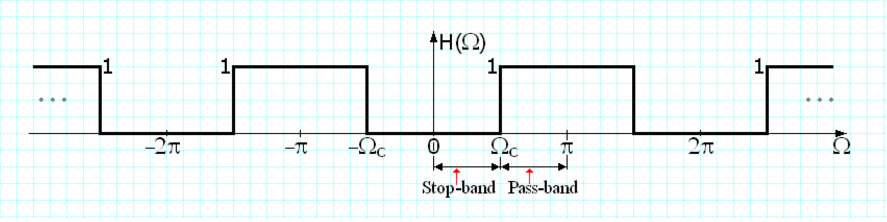
$$H(\Omega)=\begin{cases}1, & |\Omega-2k\pi|<\Omega_c,\quad k\in\mathbb{Z}\\ 0, & \text{in rest}\end{cases} \leftrightarrow h[n]=\frac{\sin\Omega_c n}{\pi n}, n\in\mathbb{Z}$$

Filtrul trece sus ideal



$$H_{TS}(\omega)=\begin{cases}0, & |\omega|<\omega_c\\ 1, & |\omega|\geq\omega_c\end{cases}=1-p_{\omega_c}(\omega)\longleftrightarrow h_{TS}(t)=\delta(t)-\frac{\sin\omega_c t}{\pi t}, \qquad t\in\mathbb{R}$$

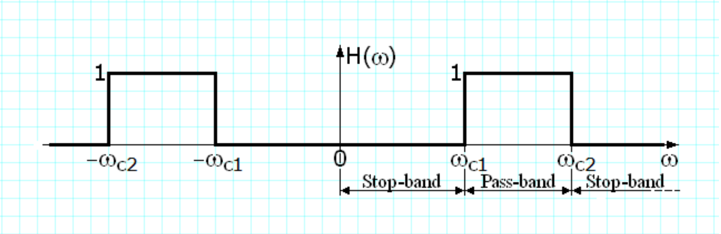
Filtrul trece sus ideal in timp discret



$$H_{TS}(\Omega)=\begin{cases}1, & |\Omega-(2k+1)\pi|<\Omega_c\\ 0, & \text{in rest}\end{cases} = 1-p_{\Omega_c}(\Omega)*\delta_{2\pi}(\Omega-\pi)$$

$$h_{TS}[n]=\delta[n]-\frac{\sin\Omega_c n}{\pi n}, \qquad n\in\mathbb{Z}$$

Filtrul trece banda ideal

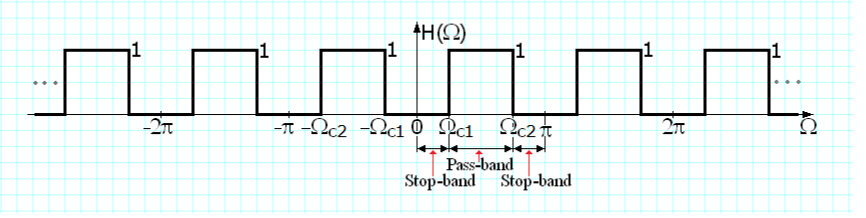


$$H_{TB}(\omega)=\begin{cases}1, & \omega_{c_1}<|\omega|<\omega_{c_2}\\ 0, & in\ \ rest\end{cases} = p_{\omega_{c1}}(\omega)-p_{\omega_{c2}}(\omega)$$

$$h_{TB}(t)=\frac{\sin\omega_{c2}t}{\pi t}-\frac{\sin\omega_{c1}t}{\pi t}$$

Doua frecvente de taiere, inferioara si superioara

Filtrul trece banda ideal in timp discret



$$H_{TB}(\Omega)=\begin{cases}1, & \Omega_{c_1}<|\Omega|<\Omega_{c_2}\\ 0, & \text{in rest}\end{cases}, \quad \Omega\in[-\pi,\ \pi)=H_0(\Omega)*\delta_{2\pi}(\Omega)$$

$$h_{TB}[n]=\frac{\sin\Omega_{c2}n}{\pi n}-\frac{\sin\Omega_{c1}n}{\pi n}$$

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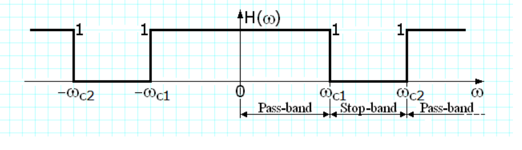
Filtrul opreste banda ideal



$$H_{OB}(\omega)=\begin{cases}0, & \omega_{c_1}<|\omega|<\omega_{c_2}\\ 1, & \text{in rest}\end{cases}$$

$$h_{OB}(t)=\delta(t)-\frac{\sin\omega_{c2}t}{\pi t}+\frac{\sin\omega_{c1}t}{\pi t}$$

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