Course 7:Phase modulation

1

Agenda

- Introduction
- Phase modulation: mathematical approach
- Demodulation of the PSK signals
- Error probability for the PSK modulation

2

Reminder

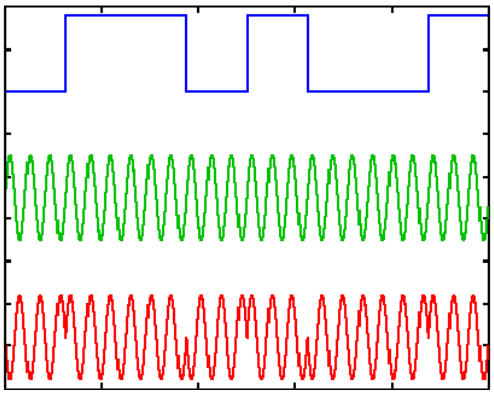
- Modulator signal: the signal to be transmitted
  - The signal can be analog or digital
  - Especially in the digital signal context, the modulator signal can be referred to as message
- Carrier signal: used to “transport” the message signal
  - Carrier signal is a sine wave (continuous wave modulation) or a periodic rectangular wave

Introduction

- Definition: PM is a method used to transmit analog or digital signals, in which the information is carried by the initial phase of a high-frequency carrier
- PM is not so popular as FM, especially for analog signal modulators (because of its complexity)
- When the modulator is digital, the frequency modulation is referred to as phase shift keying (PSK)

Graphical view

*www.pctechguide.com



PSK

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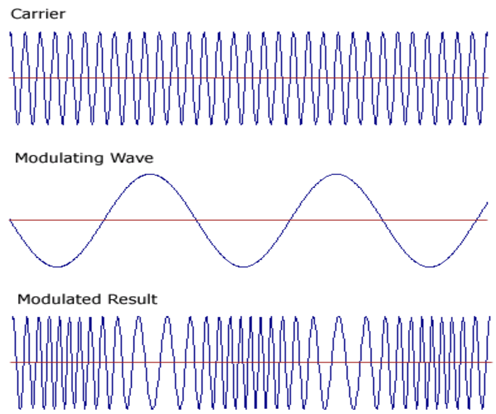


Analog PM

PM: pros and cons

Pros

- Narrow bandwidth compared to FM
- Non-coherent detection, with small performance degradation

Cons

- High complexity (especially for analog PM)

Mathematical approach

- Considering a rectangular form of the modulator signal, x(t):

$$x(t) = \sum_n a_n g(t - nT) \quad (1)$$

g(t): rectangular pulse-shaper (pulse with amplitude 1 and duration T)

a_n: data sequence to be transmitted (bits)

- If we associate a phase, Φ_n with each symbol $a_{n,}$ we get :

$$x_\Phi(t) = \sum_n \Phi_n g(t - nT) \quad (2)$$

Mathematical approach

$$x_\Phi(t) = \sum_n \Phi_n g(t - nT) \quad (2)$$

- x_Φ remains constant during the symbol interval (T)
- Φ_n is a phase value, associated with the symbol to be transmitted
- In the digital PM (PSK) we have a finite number of possible symbols, M
- M=2, corresponds to BPSK (e.g.: 90^o phase shift corresponds to “1”, -90^o phase shift corresponds to “0”)

Mathematical approach

- The transmitted PM signal is:

$$s(t) = U_0 \cos(\omega_0 t + x_\Phi(t)) = U_0 \cos(\omega_0 t + \sum_n \Phi_n g(t - nT)) \quad (3)$$

- By expanding the parenthesis above, we get:

$$s(t) = U_0 \cos(\omega_0 t) \sum_n g(t - nT) \underbrace{\cos \Phi_n}_{a_n} + U_0 \sin(\omega_0 t) \sum_n g(t - nT) \underbrace{(-\sin(\Phi_n))}_{b_n} \quad (4)$$

- Equation 4 can be re-written as:

$$s(t) = U_0 \cos(\omega_0 t) \sum_n a_n g(t - nT) + U_0 \sin(\omega_0 t) \sum_n b_n g(t - nT) \quad (5)$$

Phase modulation

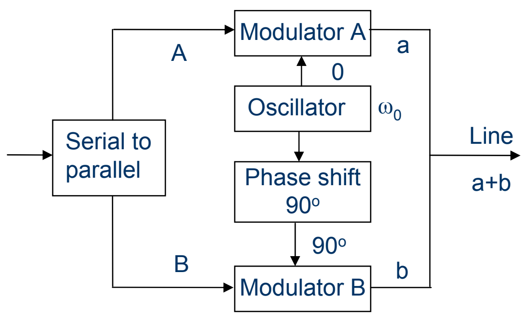
Mathematical approach

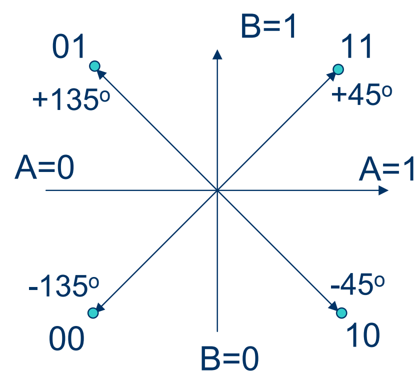
$$s(t) = U_0 \cos(\omega_0 t) \sum_n a_n g(t - nT) + U_0 \sin(\omega_0 t) \sum_n b_n g(t - nT) \quad (5)$$

- The PM signal can be seen as a sum of two amplitude modulated signals
- Two carriers are used, having the same frequency, but with a phase shift of 90^o (in-quadrature carriers)
- The main impact in the spectrum of a PM signal is given by the rectangular form of the pulse-shaping function g(t)

QPSK Modulator

- For the BPSK case, the modulation is implemented by simply inversing the sign (0º/180º phase shift)
- In QPSK (Quadrature PSK), the modulation symbol is a group of two bits

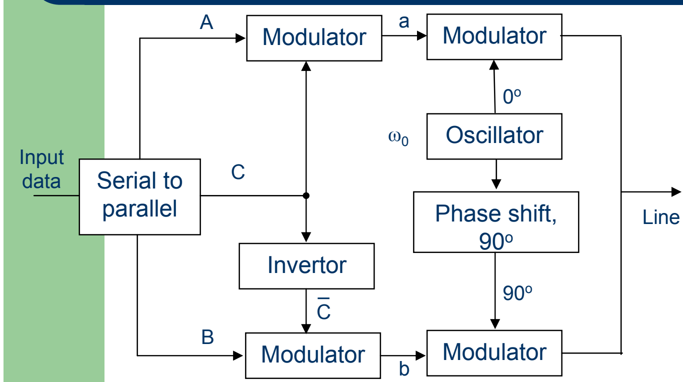


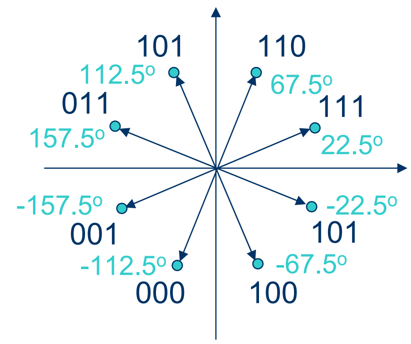


Comments on the previous scheme

- Every symbol is a group of two bits (AB)
- On the upper branch, the signal corresponding to bit "A" (e.g.:+V voltage for "1", -V voltage for "0") modulates a cosine
- On the lower branch, the signal representing the bit "B", modulates a cosine of the same frequency, but shifted by 90º
- That's the reason behind the "Q" in QPSK
- The signal from the two branches is summed an transmitted in the channel

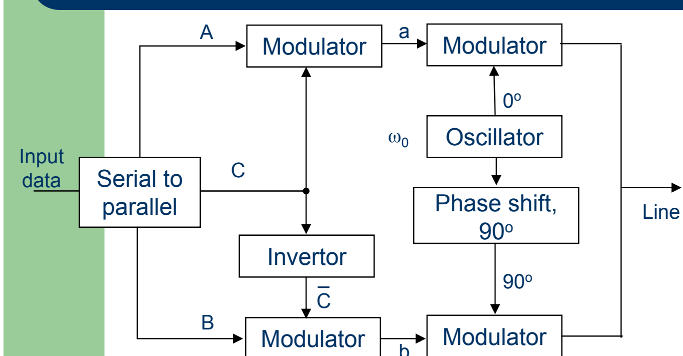
8-PSK modulator





- A group of three bits composes a symbol
- *a* and *b* will be 4-levels AM signals
- The second modulator is a phase modulator

8-PSK modulator cases

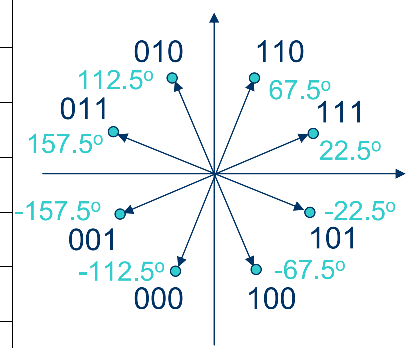


- A and B will give the sign of the signal level (a and b), whereas C establishes the amplitude
- E.g: A=1, C=1, positive sign and high amplitude of *a*, high phase shift in the modulated signal

- On the lower branch, the inverted value of C will be 0, and if B=1, we will have positive sign, low amplitude

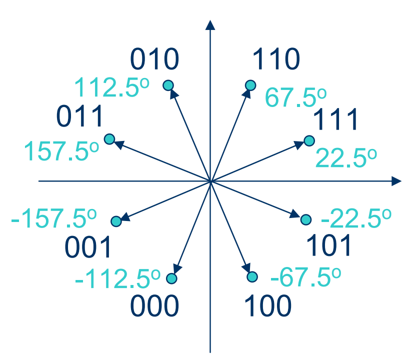
8-PSK modulator: a deeper look

A (sign)	B (sign)	C (level)	a	b	Phase
0	0	0	Negative, small amplitude	Negative, high amplitude	**-112.5°**
0	0	1	Negative, High amplitude	Negative, Small amplitude	**-157.5°**
0	1	0	Negative, Small amplitude	Positive, high amplitude	**+112.5°**
0	1	1	Negative, High amplitude	Positive, Small amplitude	**+157.5°**
1	0	0	Positive, Small amplitude	Negative, High amplitude	**-67.5°**
1	0	1	Positive, High amplitude	Negative, small amplitude	**-22.5°**
1	1	0	Positive, Small amplitude	Positive, high amplitude	**+67.5°**
1	1	1	Positive, High amplitude	Positive, Small amplitude	**+22.5°**



Comments on the previous table

- a and b are the Cartesian coordinates of the signal point
- E.g.- let’s take the message “010”: the abscissa is negative, small value, the ordinate is positive, high value
- This results in a vector whose angle with the abscissa gives the desired value (112.5º for our example)



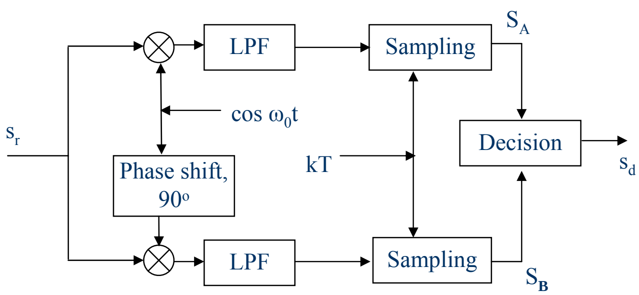
Remarks about the modulator

- The modulator schemes presented previously introduce parasite amplitude modulation
- Other techniques use a combination between phase shifting circuits (producing the desired phase) and logical circuits (selecting the desired phase according to the binary combination)
- PM can be implemented by frequency divisors too
- Differential PM is oftentimes used
- DPSK allows non-coherent demodulation

Principles

- Coherent and non-coherent demodulation
- Coherent detection
  - The phase of the received signal compared to the phase of a local reference oscillator
  - This oscillator is implemented in the receiver and is synchronized with the oscillator from the transmitter
  - Coherent detection in channel which doesn’t introduce phase variation

QPSK demodulation



Phase	S_A	S_B	s_d
+45°	+	+	11
-45°	+	-	10
+135°	-	+	01
-135°	-	-	00

•Local reference oscillator
•After the LPF, the $2\omega_0$ component is removed
•After sampling, a positive value indicates a "1", a negative sample indicates a "0" (on both branches)
•Equivalently, a single phase detector and a multi-level comparator can be used

Comments on QPSK demodulator

- The decision table from the previous slide is directly derived from the QPSK constellation (see slide 11)
- S_A and S_B are the abscissa and the ordinate values of the signal point
- E.g., if S_A is positive, A=1, no meter what happens on the lower branch
- The decision can be made independently on each branch

DBPSK Principles

- DBPSK="Differential Binary Phase Shift Keying"
- At modulator, a pre-coder provides a differential phase shift
- Instead of transmitting a_n (the initial bit sequence), the data sequence transmitted in the channel is pre-coded:

$$b_n = \overline{a_n} \oplus b_{n-1} \quad (6)$$

- Example:

n	0	1	2	3	4	5
a_n	1	1	0	1	1	0
$\overline{a_n}$	0	0	1	0	0	1
b_n	0	0	1	1	1	0

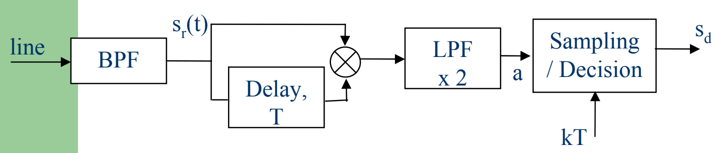
DBPSK-precoding

n	0	1	2	3	4	5
a_n	1	1	0	1	1	0
	0	0	1	0	0	1
b_n	0	0	1	1	1	0

φ_n= -90°, -90°, 90°, 90°, 90°,-90° (B law)

- The pre-coding confers differential character to the modulation
- If a_n=0, b_n will change its value compared to b_{n-1}; if a_n=1, b_n remains unchanged
- Next, PSK is applied on b_n
- The final result: whenever a_n=0, a phase shift occurs
- φ_n above correspond to the B law (A law: 0°, 180°)

DBPSK Demodulator



b_n	Transmitted signal	Phase
0	$U_0\cos(\omega_0 t)$	0°
1	$U_0\cos(\omega_0 t+180°)$	180°

- For the n-th bit, the received signal (after) BPF is:

$$s^n{}_r(t) = U_0\cos(\omega_0 t + \varphi_n), \quad (n-1)T \le t \le nT \quad (7)$$

- After the LPF (which eliminates the second harmonic), we get:

$$a = U_0\cos(\varphi_n - \varphi_{n-1}) \quad (8)$$

Remarks on the DBPSK demodulator

- The signal delay, T, introduced in the demodulator, must equal the symbol period
- The delayed signal will then correspond to the previously transmitted bit
- The signal entering the LPF will be:

$$s_p(t) = U_0\cos(\omega_0 t + \varphi_n)U_0\cos(\omega_0 t + \varphi_{n-1}), \qquad (9)$$

- By transforming this product into a sum we will obtain an oscillating component ($2\omega_0$ and a DC component, indicated by eq. 8)

DBPSK: symbol detection

- Detection is made taking into account that:

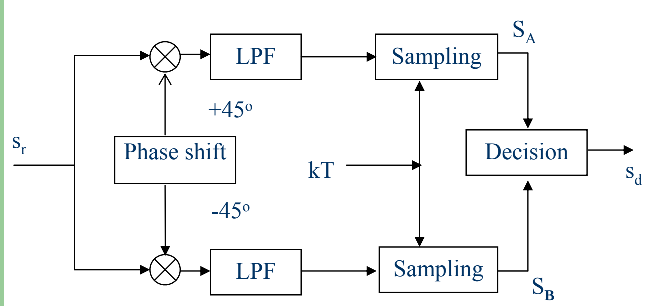
$$a_n = \overline{b_n \oplus b_{n-1}} \quad (10)$$

- The following detection table is used ("a" refers to eq. 8)

Case	$\varphi_n - \varphi_{n-1}$	a-Sign	Decision
bn≠bn-1	$\pm\pi$	-	$a_n=0$
$b_n = b_{n-1}$	0	+	$a_n=1$

- Comparison with the previous bit is implicitly made (differential detection)

DQPSK demodulation



Phase	S_A	S_B	s_d
0º	+	+	11
+90º	+	-	10
±135º	-	-	00
-90º	-	+	01

- NO Local reference oscillator needed
- After the LPF, the $2\omega_0$ component is removed
- After sampling, a positive value indicates a "1", a negative sample indicates a "0" (on both branches)

Error probability for the binary PSK