Course 6:Frequency modulation

Agenda

- Introduction
- Data transmission chain using FM
- Frequency modulation: mathematical approach
- Demodulation of the FSK signals
- Error probability for the FSK modulation

Reminder

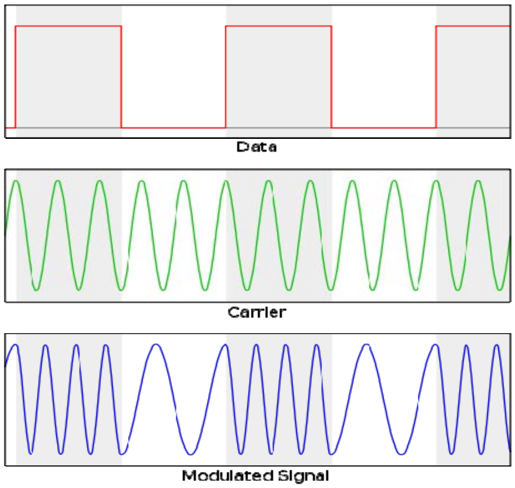
- Modulating signal: the signal to be transmitted
  - The signal can be analog or digital
  - Especially in the digital signal context, the modulating signal can be referred to as message
- Carrier signal: used to "transport" the message signal
  - Carrier signal is a sine wave (continuous wave modulation) or a periodic rectangular wave

Introduction

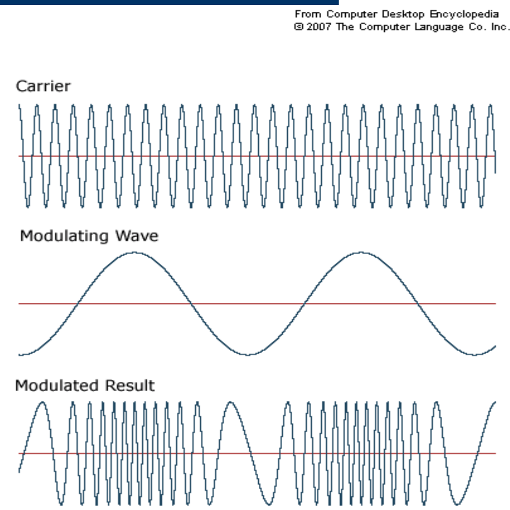
- Definition: FM is a method used to transmit analog or digital signals, in which the information is carried by the instantaneous frequency of a high-frequency carrier
- Examples: FM radio (modulating=audio signal), FM modems (modulating=digital data)
- When the modulating signal is digital, the frequency modulation is referred to as frequency shift keying

Graphical view

*From wikipedia.com

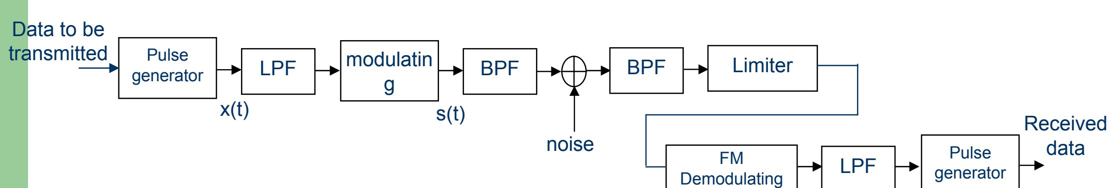


FSK



Analog FM

Data transmission chain using FM



- Transmitter
  - The pulse generator issues rectangular pulses to represent data (digital encoding)
  - LPF (Low-Pass filter): shapes (in time and frequency) the signal
  - modulating: performs FM
  - BPF (Band-Pass Filter): fits the signal to its dedicated band (to avoid interferences with the adjacent bands)
- Receiver
  - BPF: eliminates out-of-band noises and interference
  - Limiter: amplifies the received signal and generates rectangular waveforms by limiting the peaks of the modulated sine
  - FM Demodulating: demodulates the received signal
  - LPF: for baseband equalization (e.g: zero-forcing) and noise removal

Mathematical approach

- Considering a rectangular form of the modulating signal:

$$x(t) = \sum_n a_n g(t - nT) \quad (1)$$

g(t): rectangular pulse-shaper

a_n: data sequence to be transmitted

- The FM modulated signal is:

$$s(t) = U_0 \cos[\underbrace{\omega_0 t + \Delta\omega \int_0^t x(\tau) d\tau + \theta}] \quad (2)$$

Important!!!

"Instantaneous phase" φ(t)

Instantaneous frequency

- The instantaneous frequency represents the derivate of the phase:

$$\omega_i(t) = \frac{d\varphi(t)}{dt} = \frac{d}{dt}\left(\omega_0 t + \Delta\omega \int_0^t x(\tau) d\tau + \theta\right) \quad (3)$$

- Some remarks:
  - For the un-modulated carrier, the middle term of the phase φ(t) is 0 and ω_i DOES NOT DEPEND on t
  - $\omega_i(t)$ in (3) is shaped by the data signal (modulating signal)
  - If the modulating x(t) has a continuous time evolution this evolution "transfers" to the $\omega_i(t)$, giving a continuous variation of this parameter

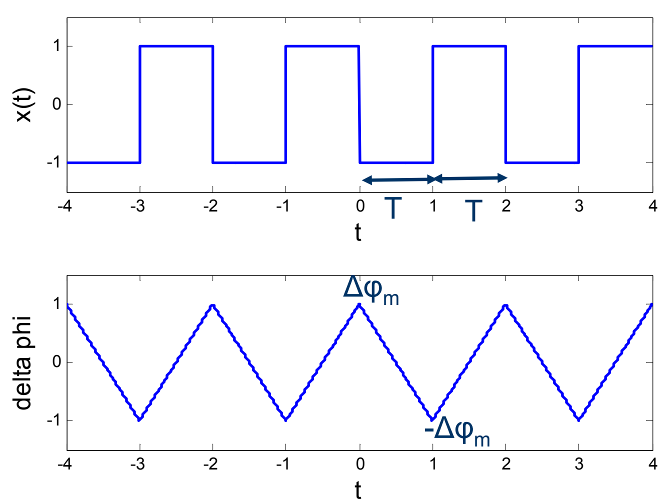
- The following notation is useful:

$$\Delta\varphi = \Delta\omega \int_0^t x(\tau) d\tau \quad (4)$$

FM types

- If $\Delta\varphi$ is continuous in time: continuous-phase FM
- If $\Delta\varphi$ has some discontinuities, we obtain FM with discontinuous phase
- The later FM has a wider PSD (consumes more bandwidth)
- Continuous phase FM is rather used in practice, especially for M-FSK, with $M \geq 4$
- In the following, only continuous-phase FM is discussed

Continuous-phase FSK



- Two symbols transmitted
- Frequencies are: $\omega_0 + \Delta\omega$ and $\omega_0 - \Delta\omega$

General remarks

- The modulating signal from the previous slide represents an alternating sequence of 1 and 0
- This generates the periodicity of the modulating signal, which is helpful as simplifying hypothesis
- Using eq. 2 and 4, the modulated signal can be expressed as:

$$s(t) = U_0 e^{j\omega_0 t} e^{j\Delta\varphi} \quad (5)$$

Key terms

- We define:

$$\Omega = \frac{2\pi}{2T} = \frac{\pi}{T} \quad (6)$$

- Ω is related to the "periodicity" of the x(t) signal, representing a kind of fundamental frequency of the modulating
- $\Delta\omega$: frequency deviation- is the maximum deviation of the instantaneous frequency compared to the carrier frequency
- $\Delta\omega$ depends on the amplitude of the modulating signal and on the modulation intensity factor (usually denoted by k_F)
- The modulation index is:

$$\beta = \frac{\Delta\omega}{\Omega} \quad (7)$$

Mathematical approach

- Due to the periodicity of $\Delta\varphi$, the modulated signal can be expressed as:

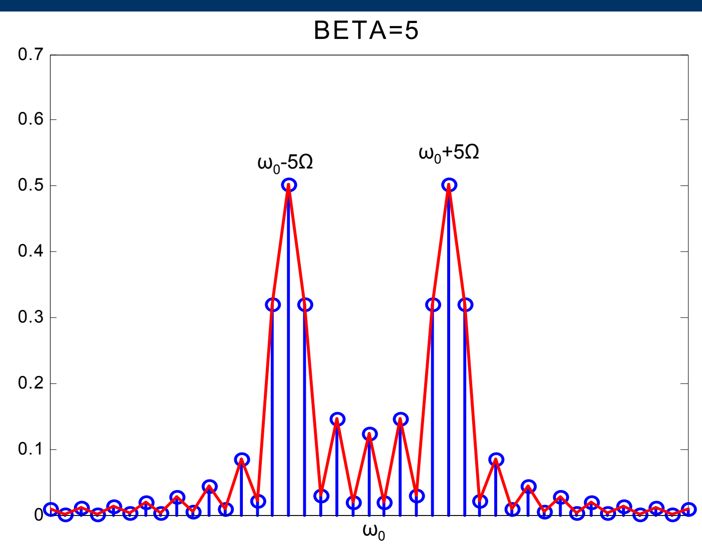
Fourier coefficients

$$s(t)=U_0\sum_n \frac{\beta}{\beta+n}\cdot\frac{\sin\frac{\pi}{2}(\beta-n)}{\frac{\pi}{2}(\beta-n)}e^{jn\Omega t}e^{j\omega_0 t} \qquad (8)$$

- U_0 and ω_0 represent the amplitude and the phase of the carrier respectively
- The FSK signal spectrum (for the case above) is made of discrete components located at ω_0, $\omega_0+\Omega$, $\omega_0-\Omega$
- The highest energy coefficients are obtained if $n=\beta$

Important!!! **These components are centered around ω_0 and separated by Ω**

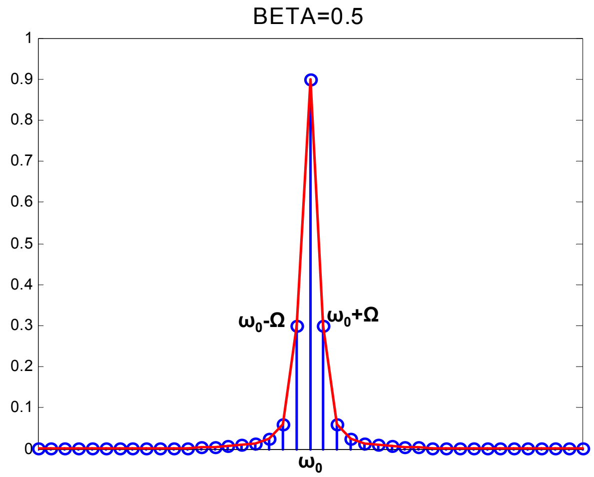
Case 1: β>>1



This is a broadband FM case !!!

- most of the energy concentrated around $\omega_0+\beta\Omega$
- large number of high energy components in the spectrum

Case 2: β<<1



This is a narrowband FM case !!!

- most of the energy concentrated around ω_0 (interval: $[\omega_0-\Omega, \omega_0+\Omega]$)
- small number of high energy components in the spectrum

Continuous phase FSK

FM signals spectrum

- Theoretically, the FM signal spectrum is infinite
- In practice, though, the energy of the components which are outside of the interval $[\omega_0+2\Delta\omega, \omega_0-2\Delta\omega]$ decrease very fast
- A “rule of thumb” given by Carson states that more than 98% of the FM signal energy lies within a bandwidth of:

$$B = 2(\beta+1)\Omega \quad (9)$$

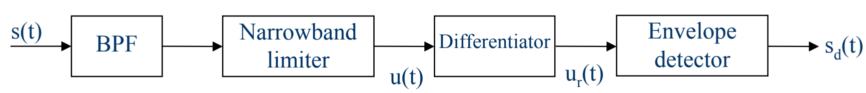
- The previous spectra were plotted for an a-priory known alternating sequence (1,0,1,0,…)
- If the data is random, the PSD obtained is very similar

Modulation: practical considerations

- When transmitting on telephone circuits (“voice modems”), the difference between the two signaling frequencies equals the maximum frequency from the modulating signal spectrum
- One of the symbols (“0” or “1”) contains exactly one supplementary oscillation period versus the other one
- If the frequency carrier is much higher than the symbol rate (e.g. 10 times higher), the best way to implement the modulating is using a VCO, directly tuned by the baseband signal

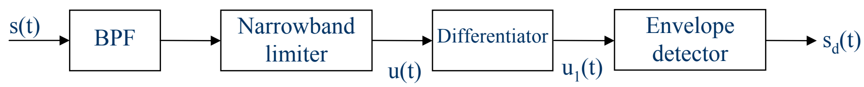
Demodulation

- Two types of demodulation: coherent and non-coherent
- A non-coherent demodulating is presented below:



- BPF: eliminates out-of-band signals (noises, interference)
- Narrowband limiter: limits the peaks of the signal and than applies a band-pass filtering around the carrier

Demodulation chain…demystified



- After limiter and BPF, the signal (identical with the transmitted signal) is:

$$u(t) = U_0 \cos(\omega_0 t + \Delta\omega \int_0^t x(\tau)d\tau + \theta) \quad (8)$$

- After the differentiator, we get:

$$u_1(t) = -K_0 U_0 [\omega_0 + \Delta\omega \cdot x(t)] \sin(\omega_0 t + \Delta\omega \int_0^t x(\tau)d\tau + \theta) \quad (9)$$

- The envelope is next detected:

$$u_1(t) = U_0 K_0 K_d [\omega_0 + \Delta\omega \cdot x(t)] = U_0 K_0 K_d \omega_0 [1 + \beta \cdot x(t)] \quad (10)$$

Threshold detector

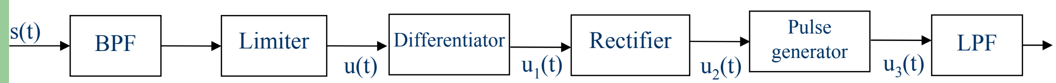
- The decision is made based on signal envelope:

$$u(t) = U_0 K_0 K_d [\omega_0 + \Delta\omega \cdot x(t)] = U_0 K_0 K_d \omega_0 [1 + \beta \cdot x(t)] \quad (10)$$

- Usually, β is small (the separation of the signaling frequencies is small)
- A threshold detector can be used to decide on the transmitted bit:

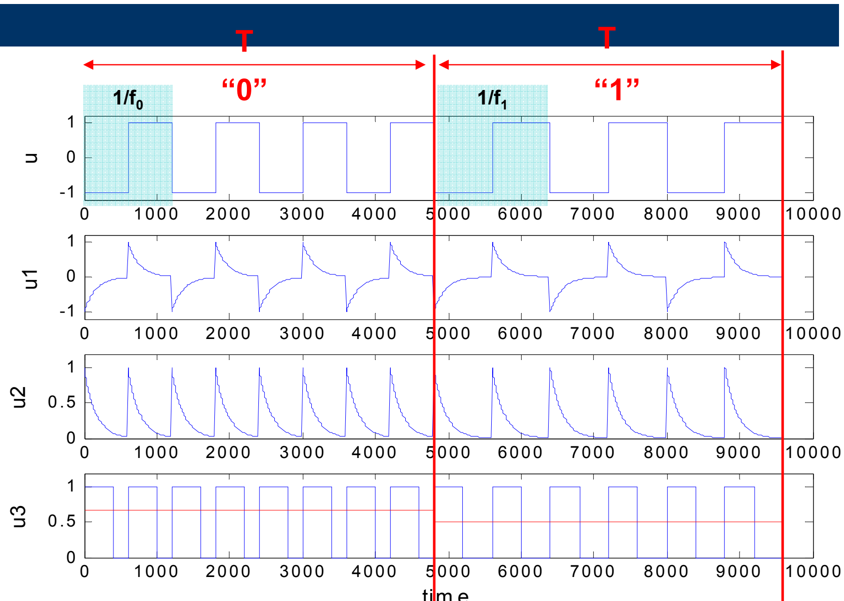
$$T = U_0 K_0 K_d \omega_0 \quad (11)$$

A different approach



- The limiter is wide-band
- It transforms the modulated sine into rectangles (trapezoidal) waveforms
- The differentiator "highlights" the transitions in these waveforms
- The signal is rectified to obtain only positive impulses
- They command a constant-duration pulse generator, which generates pulses having the same duration, but different periods given by the bit value
- LPF averages the signal over one bit duration, thus transforming the frequency information into an amplitude one

Demodulating chain



Looking behind the waveforms

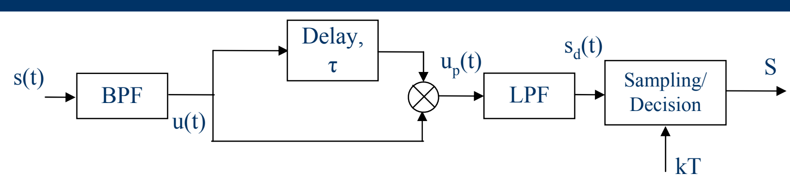
- The rectangular pulses have same duration (for “0” or “1”) but different periods
- The average voltage level, which is observed for one bit period is:

$$U_k = U_3 \frac{\tau}{T_k/2} F(0) = 2U_3 \tau f_k F(0) =$$

$$= 2U_3 \tau \frac{\omega_k}{2\pi} F(0) = U_3 \tau \frac{\omega_0 \pm \Delta\omega}{\pi} F(0) = U_3 \tau \frac{\omega_0 + \Delta\omega x(t)}{\pi} F(0) \quad (13)$$

- The pulses issued by the pulse generator must have a duration shorter than one half of the shortest period (T_0)

Differential demodulating



- The delayed version of the signal u(t) is:

$$u(t-\tau) = U_0 \cos[\omega_0 (t-\tau) + \Delta\omega \int_0^{t-\tau} x(\tau) d\tau] \quad (14)$$

- After, the multiplier, we get:

$$u(t)u(t-\tau) = \frac{U_0^2}{2} \cos(2\omega_0 t - \omega_0 \tau + \Delta\omega \int_0^t x(\tau) d\tau + \Delta\omega \int_0^{t-\tau} x(\tau) d\tau) +$$

$$\frac{U_0^2}{2} \cos(\omega_0 \tau + \Delta\omega \int_{t-\tau}^{t} x(\tau) d\tau) \quad (15)$$

How decision is made

- The LPF eliminates the $2\omega_0$ component:

$$s_d(t) = \frac{U_0^2}{2}\cos(\omega_0\tau + \Delta\omega \int_{t-\tau}^{t} x(\tau)d\tau) \quad (16)$$

- By choosing $\omega_0\tau=\pi/2$, we get:

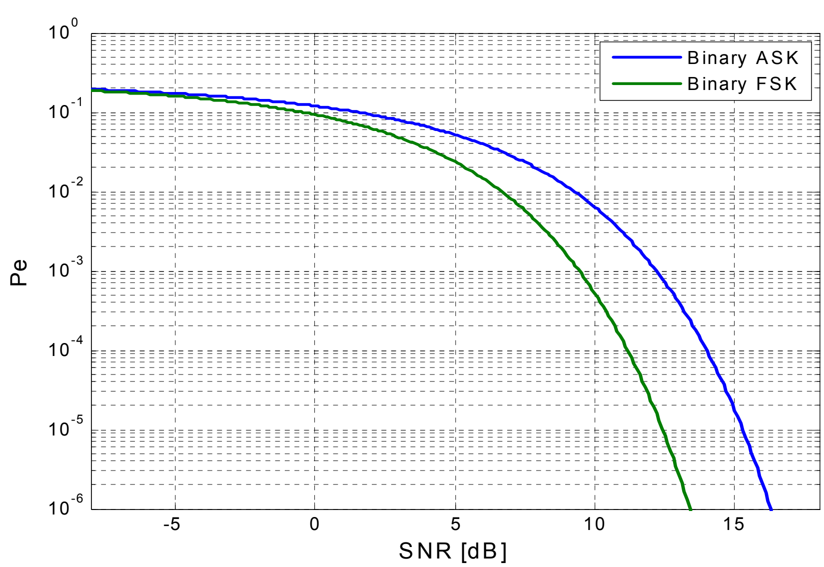
$$s_d(t) = -\frac{U_0^2}{2}\sin(\Delta\omega \int_{t-\tau}^{t} x(\tau)d\tau) \quad (17)$$

- Taking into account the binary nature of the modulation ($x(\tau)=1$ or -1), we obtain:

$$s_d(t) = -\frac{U_0^2}{2}\sin(\pm\Delta\omega\tau) = \pm\frac{U_0^2}{2}\cdot\Delta\omega\cdot\tau \quad (18)$$

Note: the signal s_d remains constant for the duration of one transmitted bit!!!

Error probability for the binary FSK



FM: pros and cons

- Pros
  - Higher resilience to additive noise and interference, compared to AM
  - Possibility to use non-linear amplifiers to amplify the FM signals
  - Non-coherent detection is an available option
- Cons:
  - Much larger bandwidth than AM (e.g. 240 KHz vs 30 KHz for a 15 KHz audio channel)