Course 11: OFDM

Orthogonal Frequency Division Multiplexing

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Agenda

- History of multi-carrier modulations
- The need for OFDM
- OFDM principles (parallel transmission, orthogonal carriers, IFFT modulator, cyclic prefix)
- OFDM: drawbacks and challenges
- OFDM in “real life”

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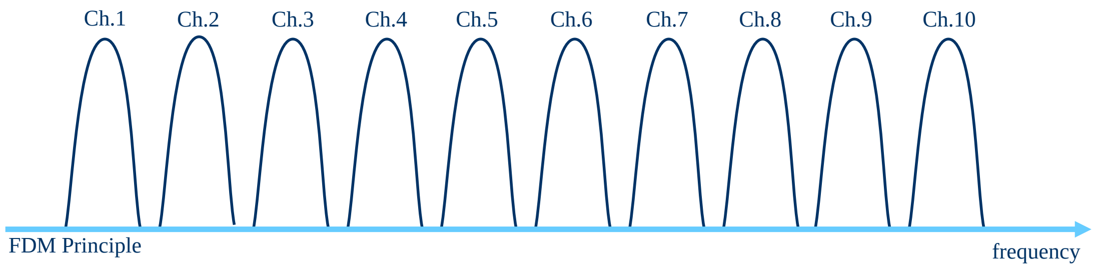
History

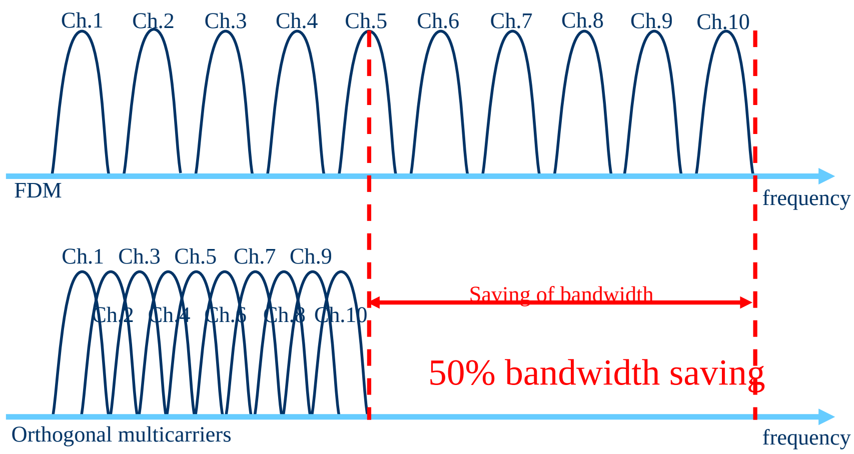
- FDM = Frequency Division Multiplexing: a way to share a single transmission channel between several users



- Every user has a dedicated bandwidth, guard intervals required
- Rather a multiple access/multiplexing technique than a modulation
- Used in PSTN, for the multiplexing of telephone calls

The step towards OFDM

- Kineplex receiver (1958): 20 orthogonal carriers are used to simultaneously convey data over a frequency selective radio channel
- A single transmitter !!!
- Bandwidth saving achieved, but prohibitive complexity
- The orthogonal carriers are very difficult to generate by analog circuits

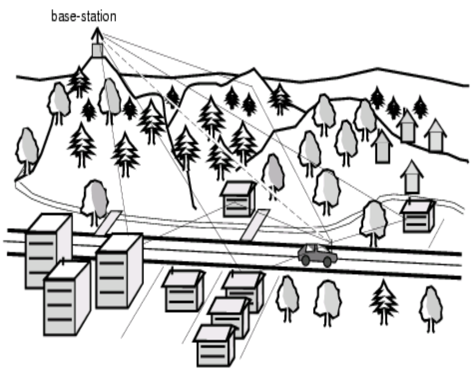


The need for OFDM

- Most of the transmission channels are frequency-selective
  - frequency components from the input signal affected differently by the channel
  - …meaning that the channel's transfer function H(f) is not flat over the bandwidth
  - this introduces Inter-Symbol interference (ISI)
  - ISI can be seen as a time-domain manifestation of the frequency selectivity
- OFDM is exceptionally robust to IIS
- Well suited to all the dispersive channels, and especially to the wireless channel

ISI: why so critical?

- In a large number of cases, the IIS is the most annoying phenomenon affecting the digital transmission
- E.g.: in a radio channel, the receiver gets multiple copies of the transmitted signal, with different delays and attenuations (=> time "spreading" of the signal)
- Example: single-carrier transmission at R_S=10Mbps
- $T_S=1/R_S=0.1\mu s$
- Multipath delay (time spreading): 10μs*



ISI for 100 symbols !!!

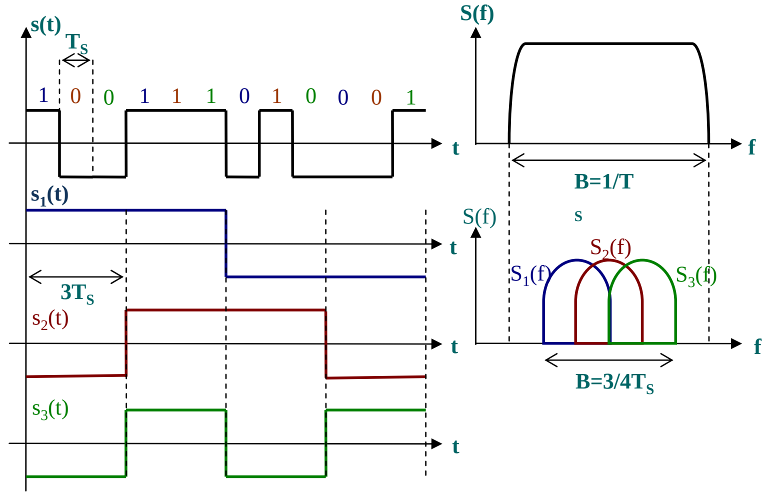
OFDM's "trick"

- The symbol duration is increased, to be much longer than the multi-path delay
- N low-rate parallel flows transmitted
  ($R_P = 1/(NT_S)$ the data rate of each flow)
  - Symbol duration: $T=1/NT_S$
  - The overall rate is preserved (like in the single carrier case): $R_S=NR_p$
- Reminder: $B \approx 1/T_S$
- E.g: if N=1000 and T_S=0.1μs, T=100 μs

The transmitted symbol is ten times longer than the multipath delay!!!

Parallel transmission on multiple carriers

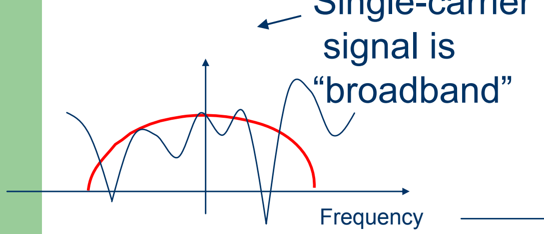
- Longer symbol duration on each parallel carrier
- The multiple carriers must be orthogonal

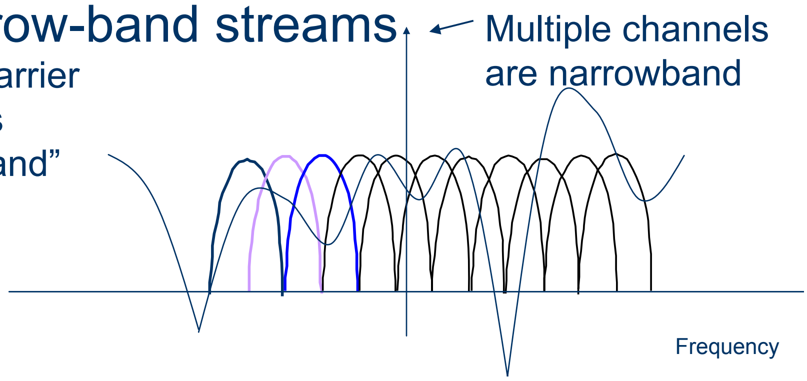


Principle of the parallel transmission on multiple carriers

Comments on the previous slide

- The direct effect of increasing the symbol duration is that its bandwidth decreases correspondingly
- In OFDM, data is transmitted through a large number of narrow-band streams





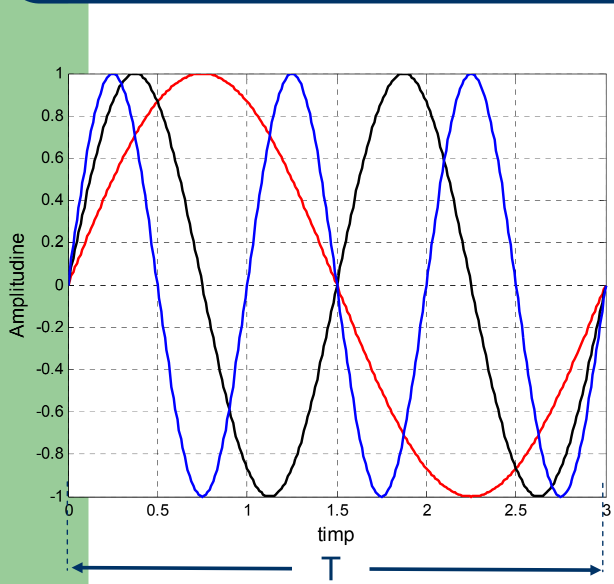
Orthogonal carriers

- The OFDM carriers are orthogonal, their frequencies being f_0, $2f_0$, $3f_0$ etc.

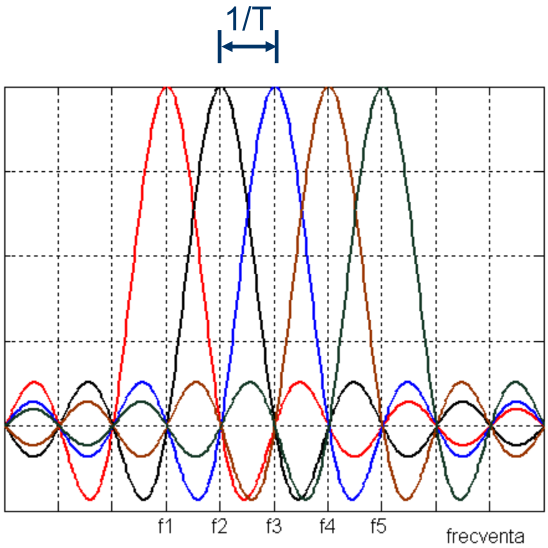
$$\frac{2}{T}\int_{kT}^{(k+1)T} \sin(mf_0t)\cdot\sin(nf_0t)\cdot dt = \begin{cases} 1, \text{ if } m=n \\ 0, \text{ if } m\neq n \end{cases} \quad (1)$$

- Complex exponentials of limited duration used in practice
  - Their duration equals OFDM's symbol time (T)
- The orthogonality is met if: $f_0=1/T$

OFDM: the carriers and their spectra



OFDM’s orthogonal carriers.



OFDM’s carriers spectra.

Comments on the previous slide

- In the time domain, every carrier covers an integer number of cycles (periods) during the symbol time (T)
  - This is a condition for orthogonality
- The carriers are time-bounded by a rectangular window, giving the symbol duration
- The sinc shape of the spectrum corresponds to a sine carrier multiplied by a rectangular time window
- At the central frequency of each carrier, all the other carriers cross zero
  - This is the orthogonality frequency view
  - Carriers must be separated by 1/T on the frequency axis

OFDM signal generation

- The signal corresponding to an OFDM symbol:

$$s_S(t) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j(\omega_n t + \varphi_n)}, \text{ pentru } \quad t \in [kT,(k+1)T] \qquad (2)$$

- If $s_s(t)$ is sampled every T_S seconds:

$$s_S(kT_S) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j\varphi_n} e^{jn\Delta\omega k T_S}, k = 0,...,N-1 \qquad (3)$$

- Inverse Fast Fourier Transform

$$g(kT_S) = \frac{1}{N}\sum_{n=0}^{N-1} G\left(\frac{n}{NT_S}\right) e^{jn2\pi k/N} \qquad (4)$$

Comments on the previous slide

- The data symbol to be transmitted (from eq. 2) is: $X[n] = A_n \cdot e^{j\varphi_n}$
  - This symbol corresponds to a certain modulation scheme (e.g. BPSK, QPSK, QAM etc)
- The signal from (2) corresponds to a single OFDM symbol (the time spanning is T)
- In equations (3) and (4), T_S is the sampling time, and it matches the duration of the serial symbol to be transmitted
- Eq. (3) and (4) are equivalent if: $A_n \cdot e^{j\varphi_n} = G(\frac{n}{NT_s}) \text{and } \Delta\omega = \frac{2\pi}{T}$
- The data symbols to be transmitted can be regarded as complex valued "frequency-domain samples"
- Eq. 4 is very close to IFFT's formula

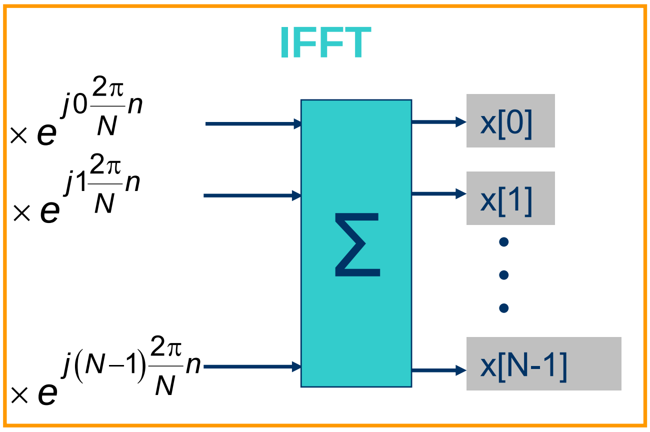
IFFT modulator

- A discrete version of the OFDM symbol is obtained by applying IFFT to the data sequence to be transmitted

X[0]   X[1]   . . .   X[N-1]

$$x[n] = \sum_{k=0}^{N-1} X[k] \cdot e^{jk\frac{2\pi}{N}n}, \quad (1.5)$$

$$n = 0, \ldots, N-1$$



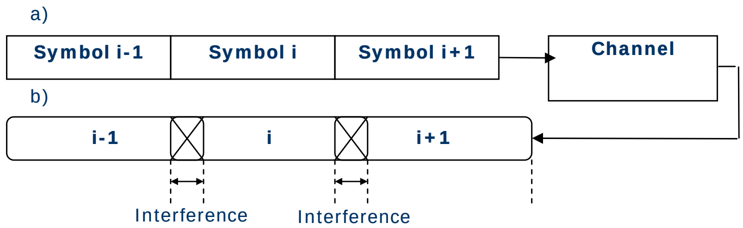
Comments on the previous slide

- FFT is a fast algorithm for the DFT implementation
- The orthogonal carriers are the complex exponentials $e^{jk\frac{2\pi}{N}n}$
- Usage of IFFT cancels the need for analog oscillators generating the orthogonal carriers
- N is the total number of (multiple) carriers used in transmission
- Demodulator uses the direct transform (FFT)

OFDM signal is generated in the baseband by signal processing only!!!

Cyclic prefix: why?

- If the channel is not ideal (is time-dispersive), the successive OFDM blocks will interfere
- This happens because each symbol is dispersed in time by the channel (see silde 6)

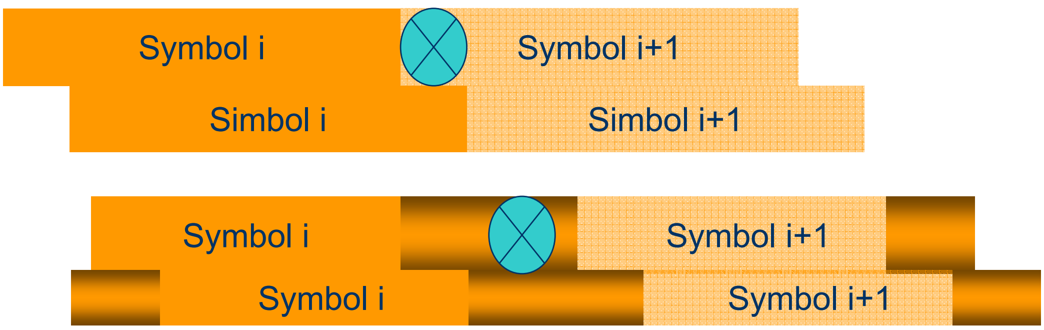


The transmission of several OFDM symbol and their interference to the receiver

- Inter-Block Interference (IBI) arises
  - Sometimes referred to as Inter-(OFDM)Symbol Interference

OFDM’s cyclic prefix

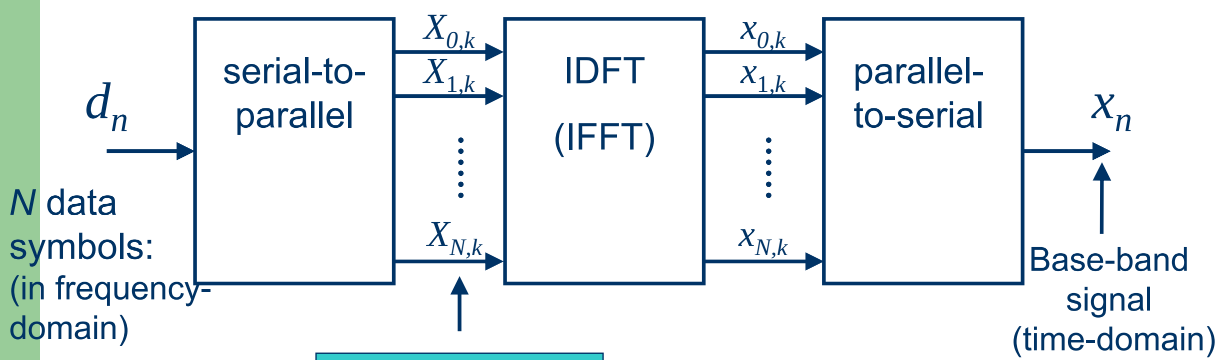
- Sometimes referred to as guard time
- Cancels IBI
- Simplifies equalization
- Eases synchronization (a signal is always transmitted)
- Cyclic Prefix is removed by the demodulator

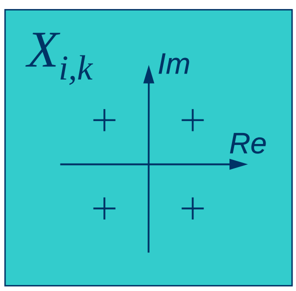


Comments on the cyclic prefix

- IBI cancellation
  - Occurs only if the CP is longer than the multipath delay
  - Longer CP duration means higher protection against ISI, but smaller efficiency
- Synchronization is simpler if a signal is always transmitted
  - E.g. a peak in the autocorrelation of the OFDM symbol could indicate the OFDM symbol start
- Equalization is simpler, because the linear convolution (x[n]*h[n]) is transformed into a periodic one
  - This allows to use a very simple, “one-tap”, frequency domain equalizer
  - Every received sample R[k], must be divided by H[k], the channel’s response at the k-th frequency line

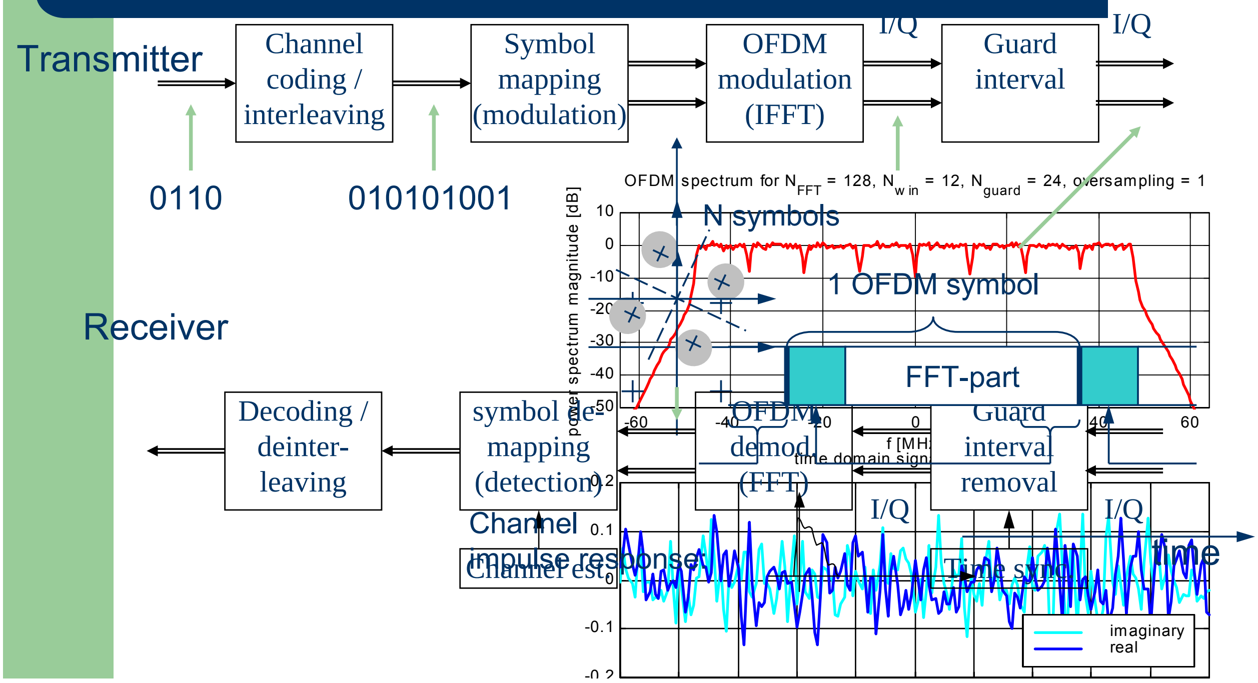
The transmitter





- k is the index of the OFDM symbol
- The base-band signal is converted to an analog signal, and modulates a high-frequency carrier before being transmitted
- N is a power of 2 (256, 512 etc)

OFDM Block Diagram



OFDM: drawbacks and challenges

- High sensitivity to time and frequency synchronization errors
  - The channel “attacks” the carriers orthogonality
- High value of Peak-to-Average-Power Ratio (PAPR) (highly variable envelope)
- Overhead due to CP
- Insufficient out-of-band attenuation

OFDM in real world

- Most of the wireless transmission standards
  - WiMAX (IEEE 802.16), WiFi (IEEE 802.11), LTE (3GPP release 8), DAVB
  - Proprietary solutions: Flash OFDM (Flarion)
- Wired broadband access
  - xDSL
- Data transmission through power line (PLC = Power Line Communications)

Acknowledgement

Some of the pictures from this slide are reproduced from the course slides published on-line by Klaus Witrisal (Signal Processing and Speech Communication Lab Technical University Graz, Austria). The title of his presentation is

Orthogonal Frequency Division Multiplexing (OFDM): Concept and System-Modeling