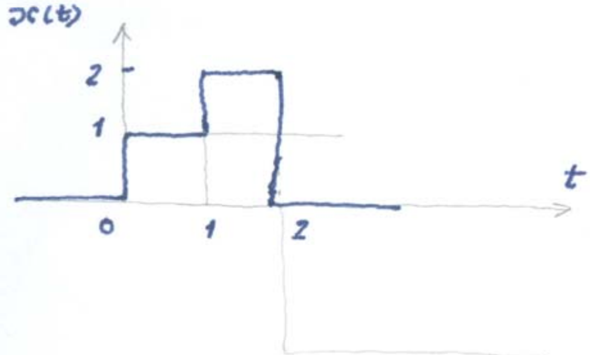
R1 (1p5)

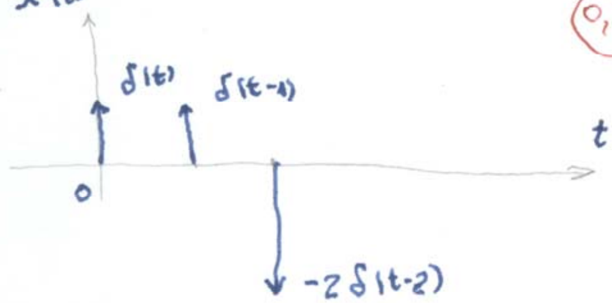
$x(t) = \sigma(t) + \sigma(t-1) - 2\sigma(t-2)$

a)



(3p)

b) (0,75p)



(0,25p) $x'(t) = \delta(t) + \delta(t-1) - 2\delta(t-2)$

Se utilizează următoarele proprietăţi ale seriilor Fourier:

P2. $c_k^{x(t-t_0)} = e^{-jk\omega_0 t_0} c_k^{x(t)}$

P3. $c_k^{x'} = jk\omega_0 c_k^{x}$

P1. $c_k^{ax+by} = a c_k^{x} + b c_k^{y}$

TP. $c_k^{\delta_T(t)} = \frac{1}{T}$

(0,5p)

$\overset{P_1}{\Longrightarrow} c_k^{x'} = c_k^{\delta_T(t)} + c_k^{\delta_T(t-1)} - 2c_k^{\delta_T(t-2)} \xrightarrow[P2,\ t_0=1 \text{ și } t_0=2]{TP,\ T=4} c_k^{x'} = \frac{1}{4} + \frac{1}{4} e^{-jk\frac{\pi}{2}} - \frac{2}{4} e^{-jk\pi}$

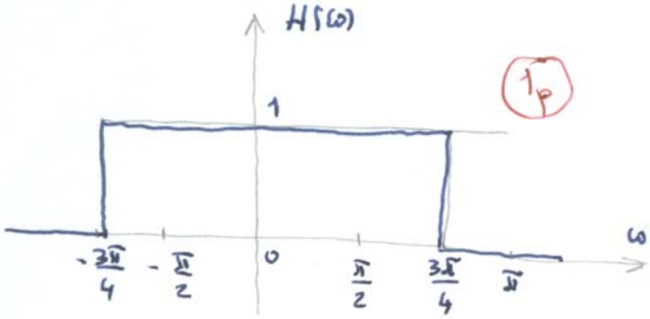
$\overset{P3}{\Longrightarrow} c_k^{x} = \frac{1}{4jk\frac{\pi}{2}}\left(1 + e^{-jk\frac{\pi}{2}} - 2e^{-jk\pi}\right)$ pt. $k \neq 0$ $\Longleftrightarrow$ (1p)

$\Longleftrightarrow c_k^{x} = \frac{1}{2jk\pi}\left(1 + \cos\frac{k\pi}{2} - j\sin\frac{k\pi}{2} - 2\cos k\pi + 2j\underbrace{\sin k\pi}_{0}\right)$ pt. $k \neq 0$

$$\boxed{c_0^{x} = \frac{1}{4}\int_0^2 x(t)\,dt = \frac{1}{4}(4-1) = \frac{3}{4};\quad c_k^{x} = \begin{cases} \frac{(-1)^p - 1}{4jp\pi}, & k = 2p,\ p \neq 0 \\ \frac{3 - j(-1)^p}{(4p+2)j\pi}, & k = 2p+1 \end{cases}}$$

(0,5p)

c)



(1p)

$$c_k^y = c_k^x H(k\omega_0) = c_k^x H\left(k\frac{\pi}{2}\right)$$

Dar $H\left(k\frac{\pi}{2}\right) = \begin{cases} 1, & k = -1, 0, 1 \\ 0, & \text{în rest.} \end{cases}$

Deci pentru $k < -1$ și $k > 2$ $c_k^y = 0$

(0,5p)

$$c_{-1}^y = c_{-1}^x = \frac{3+j}{-2j\pi} = \frac{-1+3j}{2\pi} = \frac{\sqrt{10}}{2\pi} e^{j(\pi - \operatorname{arctg} 3)}$$

$$c_0^y = c_0^x = \frac{3}{4}$$

$$c_1^y = \frac{3-j}{2j\pi} = \frac{-1-3j}{2\pi} = \frac{\sqrt{10}}{2\pi} e^{j(\pi + \operatorname{arctg} 3)}$$

(1p)

$$y_p(t) = \sum_{k=-1}^{1} c_k^y e^{jk\frac{\pi}{2}t} = \frac{\sqrt{10}}{2\pi} e^{j(\pi - \operatorname{arctg} 3)} \cdot e^{-j\frac{\pi}{2}t} + \frac{3}{4} +$$

$$+ \frac{\sqrt{10}}{2\pi} e^{j(\pi + \operatorname{arctg} 3)} \cdot e^{j\frac{\pi}{2}t} = \frac{3}{4} + \frac{\sqrt{10}}{2\pi}\left[- e^{-j\left(\frac{\pi}{2}t + \operatorname{arctg} 3\right)} + \right.$$

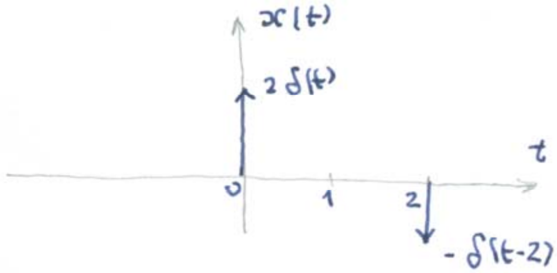
$$\left. + \left(- e^{j\left(\frac{\pi}{2}t + \operatorname{arctg} 3\right)}\right)\right] = \frac{3}{4} - \frac{\sqrt{10}}{\pi} \cos\left(\frac{\pi}{2}t + \operatorname{arctg} 3\right)$$

Deci: $y_p(t) = \frac{3}{4} - \frac{\sqrt{10}}{\pi} \cos\left(\frac{\pi}{2}t + \operatorname{arctg} 3\right)$. (0,5p)

R2

$x(t) = 2\delta(t) - \delta(t-2)$

a)



3p

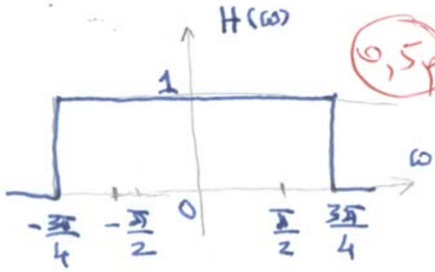
b) $c_k^x = 2c_k^{\delta(t)} - c_k^{\delta(t-2)}$ 0,5p ; $c_k^{\delta(t-2)} = e^{-jk\frac{2\pi}{4}2} c_k^{\delta(t)} = e^{-jk\pi}\frac{1}{4}$ 0,25p

$c_k^x = \frac{2}{4} - \frac{1}{4}e^{-jk\pi} = \frac{2}{4} - \frac{1}{4}(-1)^k = \begin{cases} \frac{1}{4}, & k-\text{par} \\ \frac{3}{4}, & k-\text{impar} \end{cases}$ 1,25p

$x_p(t) = \sum_{k=-\infty}^{\infty} c_k^x e^{jk\frac{\pi}{2}t} = \sum_{p=-\infty}^{\infty} c_{2p}^x e^{jp\pi t} + \sum_{p=-\infty}^{\infty} c_{2p+1}^x e^{j\frac{(2p+1)}{2}}$ 0,5p

$x_p(t) = \frac{1}{4}\sum_{p=-\infty}^{\infty} e^{jp\pi t} + \frac{3}{4}\sum_{p=-\infty}^{\infty} e^{j\frac{(2p+1)\pi t}{2}}$ 0,5p

c)



0,5p

$c_k^y = c_k^x H(k\omega_0) = c_k^x H(k\frac{\pi}{2})$

Dar $H(k\frac{\pi}{2}) = \begin{cases} 1, & k=-1, 0, 1. \\ 0, & \text{în rest.} \end{cases}$ 0,5p

De aceea $c_k^y = 0$ pt. $k < -1$ sau $k > 1$ 0,5p ; $c_{-1}^y = c_{-1}^x = \frac{3}{4}$ 0,5p ; $c_0^y = c_0^x = \frac{1}{4}$ 0,25p

$c_1^y = c_1^x = \frac{3}{4}$ 0,25p

$y_p(t) = \sum_{k=-1}^{1} c_k^y e^{jk\frac{\pi}{2}t} = \frac{3}{4}e^{-j\frac{\pi}{2}t} + \frac{1}{4} + \frac{3}{4}e^{j\frac{\pi}{2}t} = \frac{3}{2}\cos\frac{\pi}{2}t + \frac{1}{4}$

$y_p(t) = \frac{1}{4} + \frac{3}{4}\cos\frac{\pi}{2}t$ 0,5p

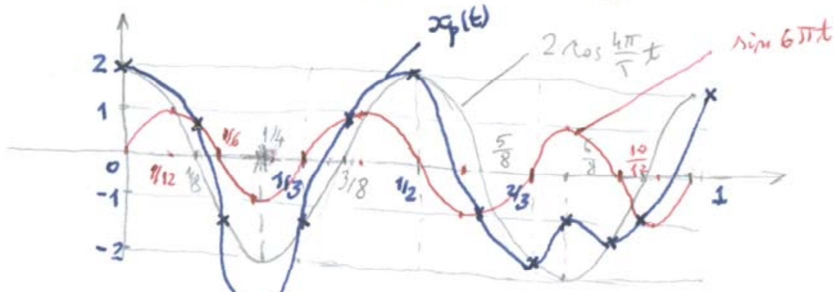
R3. 1,5p

$x_p(t) = 2\cos 4\pi t + \sin 6\pi t$

a) $x_p(t) = 2\cos \frac{2\pi}{\frac{1}{2}} t + \sin \frac{2\pi}{\frac{1}{3}} t$ 0,25p

$T_1 = \frac{1}{2}$; $T_2 = \frac{1}{3}$; $T = c.m.m.m.c\left(\frac{1}{2}, \frac{1}{3}\right) = 1$ 0,25p

$x_p(t) = 2\cos 2 \frac{2\pi}{T} t + \sin 3 \frac{2\pi}{T} t$ 0,5p



2p

$\frac{1}{12}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{7}{12}$ $\frac{8}{12}$ $\frac{9}{12}$ $\frac{10}{12}$ $\frac{11}{12}$

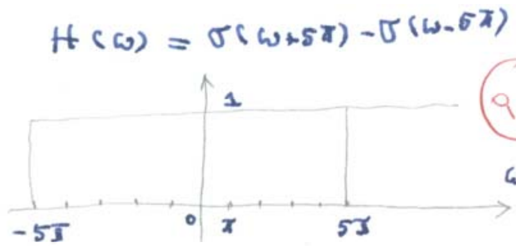
$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$

b) $x_p(t) = 2\cos 2\cdot 2\pi t + \sin 3\cdot 2\pi t = 2\,\frac{e^{j2\cdot 2\pi t} + e^{-j2\cdot 2\pi t}}{2} + \frac{e^{j\cdot 3\cdot 2\pi t} - e^{-j3\cdot 2\pi t}}{2j} =$ 1p

$= -\frac{1}{2j} e^{-j3\cdot 2\pi t} + e^{-j\cdot 2\cdot 2\pi t} + e^{j\cdot 2\cdot 2\pi t} + \frac{1}{2j} e^{j\cdot 3\cdot 2\pi t}$ 1p

$c^x_{-3} = -\frac{1}{2j}$; $c^x_{-2} = 1 = c^x_2$; $c^x_3 = \frac{1}{2j}$ 1p

c) $H(\omega) = \sigma(\omega + 5\pi) - \sigma(\omega - 5\pi)$



0,5p

$c^y_k = c^x_k \cdot H(k\omega_0) = c^x_k H(k2\pi)$

Dar $H(k2\pi) = \begin{cases} 1, & k = -2, -1, 0, 1, 2 \\ 0, & \text{în rest} \end{cases}$ 0,5p

De aceea $c^y_k = 0$ pt. $k < -2$ sau $k > 2 \Rightarrow c^y_{-3} = c^y_3 = 0$; 1p

0,5p

$c^y_{-2} = c^x_{-2} = 1$, respectiv $c^y_2 = c^x_2 = 1$. În consecință:

$y_p(t) = e^{-j2\cdot 2\pi t} + e^{j2\cdot 2\pi t} = 2\cos 4\pi t.$ 0,5p